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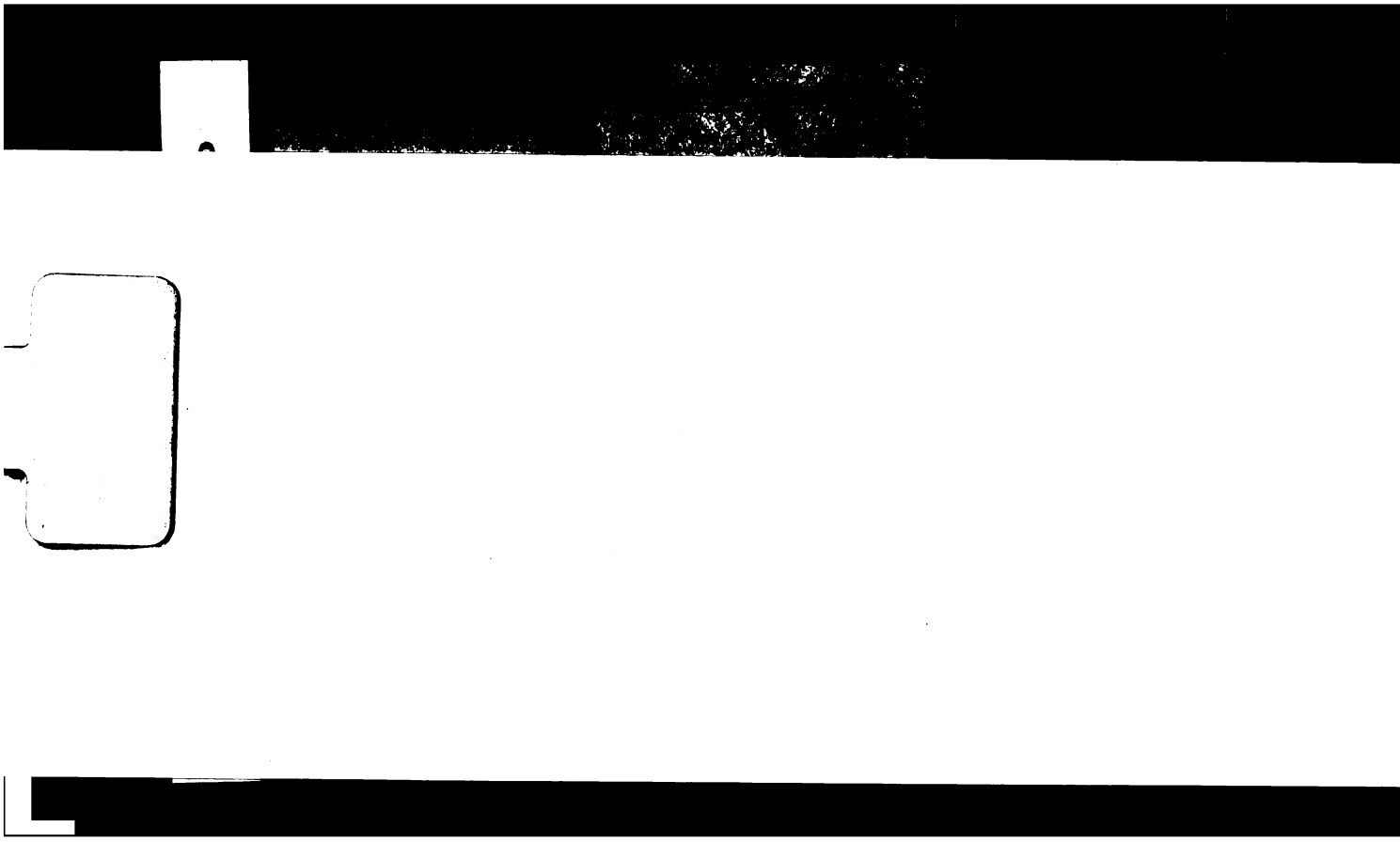

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STEAM

AND

THE MARINE ENGINE.

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THE STUDY OF

STEAM

AND

THE MARINE ENGINE.

FOR YOUNG SEA OFFICERS IN H. M. NAVY
THE MERCHANT NAVY, ETC.

BEING A

COMPLETE INITIATION INTO A KNOWLEDGE OF PRINCIPLES AND THEIR APPLICATION TO PRACTICE.

BY

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AUTHOR OF "PROJECTION AND CALCULATION OF THE SPHERE" ETC.

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PREFACE.

A BOOK is wanted in which the principles of steam are explained in a manner adapted to young officers in Her Majesty's Navy and the Merchant Service.

The very completeness of some of the many really valuable works already published (so far as regards the treatment of certain subjects), is an obstacle to their use in elementary studies, because the information sought is scattered through so many pages, that a difficulty arises in the ready selection of matter fit for the educational purposes of judicious training.

Herein is provided a course of study adapted either to those who have a knowledge of the engine itself, but have had no time to study principles, or to those who know little or nothing of the engine or of principles, but who desire an acquaintance with both.

Those whose schoolboy days have long passed, but whose minds have been healthily exercised in vigorous active duties, require a peculiar measure of help in the pursuit of that knowledge which is essential to their proficiency.

Various opinions exist as to the best mode in which to educate for the profession of an engineer. The writer's

opinion, after very many years' experience in teaching them, is, that in their case we cannot consistently adopt the ordinary routine of school system; — because, in the first place, a thoroughly good and useful officer of large *practical* experience, but who is ignorant of the higher principles of science, *may* do credit to his profession, although it is certain that such an officer, if *better* educated, would do *very much more* credit; and secondly, because time can scarcely be spared by them for elaborate investigation of principles, until some favourable opportunity occurs *after* having actually entered upon their duties afloat. It is then in the judicious direction of the *earlier* studies, that the best scholastic advice, and a book carefully prepared for the purpose, are needed by the class referred to: not, indeed, for them only, but *such a work as will be useful to sea officers in Her Majesty's Service, and the mercantile marine generally.*

That the public judgment will be freely called into exercise upon the consistency of the author of such a book is probable; for he must relinquish (for a time) many conventional points of routine, and advise with an independence which involves great professional responsibility; such, indeed, as to the reputation of the mere teacher or the mere engineer, might be hazardous. But be it so — the course has been successfully tested. *It cannot really be inconsistent to condense information, and to lighten labour, so long as the main object, that of clearness, and a proper illustration of principles, be kept in view.* A certain amount of drudgery, which is supposed to be essential to the progress of the heedless boy, cannot be necessary for the thoughtful, mature, and earnest aspirant who, already in

the service of his country, looks to his future as dependent on his professional acquirements. And yet what distinction have educational works as yet recognised?

We all know that elementary study after our school-days have passed is a long and dreary railway of monotony, but when we *can* scatter about the occasional "stations," or necessary resting-places, a few little patches of flowers, and glimpses of higher cultivation, these are the little *oases* which refresh, and the reliefs which strengthen the mind and purposes for increased progress. Occasional peeps, therefore, into the higher branches of study will be allowed in the course of the work; and thus, in our voyage of discovery (as it were), we shall often take *soundings*, when the not very clear waters in which we are sometimes obliged to sail will hide the bottom from our eyes. That course of study is the best, which keeps us the nearest to principles, while we are actively engaged with practical illustration. For the main distinction between the man of science and the mere smatterer lies in their comparative knowledge and appreciation of the "Why and Wherefore."

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THE MARINE ENGINE.

PART I.

THE PROFESSIONAL REQUIREMENTS OF NAVAL ENGINEERS.

THE operations of the engineer are so multifarious, that to condense a description of his required work of study within moderate limits is difficult, yet the author feels that his long experience can satisfy the requirements of the numerous students of steam, who feel that their newly developed necessities have been in some measure overlooked by teachers; at least by those whose pens might with so much advantage have recorded the instructive lessons which their tongues so untiringly, and so profitably to their hearers, have uttered in their class-rooms.

Preparation for the sciences involved in the studies of an engineer has hitherto been left almost exclusively to the varying opinions of the multitude of teachers, who, for the most part, have little beyond theoretical knowledge of the matter they work upon: hence many still valuable books are not found equal to the needs of those who use them. This induces a thoughtless, "plodding" habit of study, with a vague trust in the effect of *quantity* rather than of the *quality* of such study, instead of causing a lively relish for proofs of daily and hourly progress.

Engineers are a class unlike most others. The soldier or the sailor officer need not, of necessity, rise from "the ranks" or the forecastle; but the very importance and responsibility, and the very nature of the work of an engineer, demands, *whatever be his social position*, an apprenticeship at an early period of life, and a personal sacrifice to manual labour, which during such apprenticeship leaves little time for the cultivation of the mind. Unless a youth at school some years previously, have had his time and talents devoted to special preparation, the completion of the education of an engineer (if such ever *can* be completed) must be the work of early manhood.

Since the institution, therefore, of public competition, the young engineer has been placed in an unfavourable position, for in such examinations the most consummate skill in mechanism is not considered. In his case, therefore, book-learning is likely to be unfairly estimated by the general public. So much more, then, do engineers demand the best assistance which teachers can give them.

It cannot be denied that their practical knowledge is a first essential. Without this such men as Watt, Stephenson, Telford, Brunel, Rennie, Scott Russell, Napier, Penn, Boulton, Maudslay, Lloyd, Armstrong, and a hundred others of European celebrity, could never have attained their eminence; but it must also be specially remembered that if they toiled at the bench, or the anvil, or the lathe, *it was the experience gained there* which laid the foundation of their fame as men of science; for it was on this practical knowledge only that they could possibly have arranged the materials gathered with no less toil and perseverance from the repertories of science, and with them have formed the accomplished engineer. Such eminence could never have been attained without solid knowledge of principles. With this knowledge a man becomes an "engineer;" without it he is only a skilled workman, nor could any amount of

gold lace make him otherwise: but it must be remembered that this mere "knowledge of principles" is attainable by any student, however unskilled in practical mechanics, so that the theoretical deficiencies of an engineer, where such exist, are not difficult of detection even by those who never entered an engine-room — a powerful reason why no time should be lost, when opportunity does offer, in entering the pleasant avenues of science, and lading oneself with its fruits. But difficulty has hitherto arisen in the gathering of such fruit, from ignorance as to a proper selection of it. Let this book then be a guide; it will not gather it for the student, but it will direct him to parts of the vast tree of knowledge whence it may be *easily* plucked, and will even occasionally bend down within his reach a well-laden branch.

The word "easily" is not used to intimate that no trouble is to be taken, for knowledge is essentially the reward of the diligent; but attention to the hints contained in this little book *will turn such diligence to the best account.*

It is a question whether the satisfaction felt after the exercise of such diligence can be exceeded by any other kind of personal self-approval whatever; for to owe one's position to one's own exertions is a constant source of pleasant reflection: and such characters are warmly appreciated by the community as they deserve to be.

The author has a thorough knowledge of the class for whom he writes. For many years he has led them along the steeps and crags of study, and his arm has supported many an aspirant (who was before unconscious of his own mental strength), until he reached, with a firm step, the solid ground of proficiency. He claims, therefore, the privilege of plain speaking while he adds a few remarks upon what demand the country reasonably makes on those who enter her Majesty's service as officers of engineers,

and upon what the proficiency of a naval engineer ought to be.

The very nature of sea service calls for superior intelligence in those on whom depend the care and management of a ship's machinery. A new era has reached us. The last of our sailing line-of-battle ships has been paid off in port. Their successors, the iron-clad "Warriors" and "Defences," seem indeed to belong to an essentially Iron Age, and call for high attainments in their engineers. The *prestige* of the British fleet has to be upheld in future by a partially divided responsibility,—for whatever may be the nautical skill and bravery of the military executive of a fleet, its usefulness or efficiency as an arm of war will rest very much on the activity and experience of those officers whose frequently perilous avocation is in connection with work involving extensive mechanical, mathematical, and philosophical questions. All these may in time of war be called into *sudden* requisition by any emergency, such as (for instance) the entry of a shot or shell into an engine-room. It is then that the courage of the man, with the coolness of the philosopher, the practised hand of the mechanic, the ready resource of the cultivated and habitually observant mind, will be seen in emulation of the equally undaunted bravery and activity of the naval warrior on the decks above.

Success in warfare, then, must in future be attributed jointly to these powers as acting in perfect harmony. It has not yet been publicly conceded that these two branches of officers need the same elementary groundwork of education. But they do; and no reason can be adduced why preparation for the one may not in future be made available for both. As an instance, change the names of the sides and angles of a triangle, and the one study merges into the other. The requirement in each case is similar; occasions may occur on board ship when it may be convenient

to accept the services of the one in aid of the other's department, although the two branches of naval education appear to be widely different.

The naval executive officer is responsible for the trim of the sails, the correctness of the course steered, the general discipline of the ship, &c., but should a spar or a mast receive a shot or damage, its reparation or replacement is delegated to officers of inferior rank; but far different is it with the responsibilities of the naval engineer. Damage to the machinery is required to be promptly remedied by the personal labour of the most skilful, whoever he may be; and there is scarcely a nook of science from which the engineer may not be required to draw his resources, and that suddenly.

Such minds, then, require cautious and careful preparation. The engineer should be, and will shortly be, in a scientific point of view, the best educated man in the ship. It is not necessary here to notice the extent to which this is appreciated in those of foreign nations.

The naval cadet has advantages over the young engineer in being allowed certain educational helps before actual entry into the service. But it is difficult as yet to offer the same advantages to young engineers. They come direct from the factory; and although a floating college has been established at Sheerness for their assistance in study while in the steam reserve, yet the difficulty of allowing a fixed period of regular attendance, were it only for three months to each, has been severely felt and regretted by some three hundred officers, who have eagerly seized upon every opportunity of benefiting by the means of study offered in the "Devonshire;" and not only has a proper spirit been manifested by these young officers, but the author's labour among them has been a source of great pleasure and satisfaction to him, as no class of young men could possibly have behaved with more uniform and

thorough propriety and attention in all respects. This public tribute of approval and thanks is due to them; may it prove a stimulant towards professional prosperity.

On entering the navy a young engineer soon appreciates the exigencies of his new duties; and the thought of a bright future in his professional career as an officer in her Majesty's service, as soon, in many cases, suggests the probable, though unavoidable, deficiencies in mental cultivation, which he at once resolves to supply. It is precisely this stage of his progress at which a work on educational training, such as this is intended to be, seems to be called for. He needs a friendly teacher, not merely to help him through his work, but to suggest the course of work itself. He will probably see his messmates, some with Tate's, others with Jeans's excellent little treatises, while some are flogging and "grinding" at Colenso, Bourne, Main, and Brown, and various other works of great intrinsic value; but the numerous paths of deliberate systematic study are a labyrinth to a beginner, which he too often hesitates to enter, until accident or companionship drifts him into a course, which he enters upon doubtfully, and therefore without that energy which springs from conviction that he is marching along the *best road* to his destination, and under the best guidance.

Naval officers of the executive will also be ready to refresh their memories in science, if a judiciously condensed work on the subject of steam be available.

HINTS ON THE STUDY OF STEAM.

It must be borne in mind, that while the following arrangements are offered as such a guide as is needed, the work points out rather the order in which subjects are to

be taken, than gives a complete course of study of each subject *in the customary detail.*

For a work of this description it is necessary that the course be curtailed to the very limits of prudence, in order to meet the full wants of those who have little time to devote to "book-work." Yet it is presumed that a careful attention to what is herein said, will do even more than lay a solid foundation for the carrying out the more elaborate studies of the engineer or naval officer. As an instance of the curtailment referred to, in treating of the mechanical powers, the wheel and axle are barely mentioned as such; but the fundamental principles, such as the lever, and the so extensively applicable parallelogram of forces, are carefully explained; and inasmuch as the strength of a ship depends on the quality and solidity of her framework, and not upon the decorative fittings, so is the proper education of an engineer dependent on clearly explained general principles rather than on elaborate polish on two or three popular subjects.

The task of self-examination is not difficult when once the mind has been impressed with the indispensability of study. Above all things, a certain facility in calculation is desirable. A few miscellaneous questions from any good work on arithmetic will test this. If vulgar fractions be thoroughly understood, and decimals are properly ready for use when wanted, the student may proceed onwards in this work to "Ratios" (page 11). If, however, fractions have not really been mastered, it matters not how many times a student may have "gone through" them in ordinary school works on arithmetic: let him lose no time in resuming the study. His want of success may be attributed, in many cases, to the book from which he worked. It is worth notice, that all elementary books on arithmetic, which have their answers on the same page as the question, are objectionable, as not only does a sight of the

answer frequently lead to the solution, but a feeling of self-reliance is, in such cases, necessarily prevented. Perhaps no works on arithmetic have appeared in which a feeling of independence is imperceptibly generated with such success as in Hind's or Colenso's Arithmetic and Algebra, and one of these (especially the latter) cannot be too soon consulted. Vulgar fractions are therein especially well explained, and not a phrase nor a word should be passed over heedlessly, and the more the student is pinched for time, the more pressingly is this advice appropriate. The above applies also to his decimal arithmetic; concerning which, however, it may be noticed, that as engineers, in their ordinary work, have to deal with units of small magnitude, such as feet, inches, &c., and as the third decimal place will give him accuracy to the thousandth part, a quantity scarcely appreciable in practice, the student need not *at first* spend much time over repeating decimals (a knowledge of the *nature* of them is however desirable), until time can be spared to carefully run through them. Too many overlook the pages of Colenso which explain the least common multiple: proper attention to this saves immense labour in general practice. On becoming, therefore, well acquainted with fractions, the student will proceed to "Ratios" (page 11).

No one can prepare a book of this description without some hesitation, because, as already observed, such admirable works are already before the public; but let any one who is ignorant of steam machinery attempt to commence the study, and he will soon be led to suspect that the minds of accomplished engineers, who are thoroughly masters of their science, and who can criticise the minutest details of an engine with professional accuracy, have but little sympathy for those who are really ignorant of the whole subject. It is the lot of many to have no knowledge whatever of the steam-engine, having been without

the means of conveniently attending either an engine-room or a lecture-room. Many will have already opened books in search of assistance, but will most probably have become bewildered and discouraged. The author recently witnessed the efforts of a person to glean information on the subject of the cylinder, it being the most important part of the engine; this person had turned to a highly popular and (in many respects) really valuable book for such explanation, but it seems almost incredible that, although the table of contents numbers hundreds of headings and subjects, the highly important words, "cylinder," "indicator," &c., do not occur therein. It is therefore an *educational work, comprehending brief but adequate and connected descriptions that naval engineers, and indeed naval officers of all grades, are asking for*—a work adapted to the wants of "beginners."

Imagine a young landsman, desirous of learning something of seamanship, being put, as his first lesson, on the deck of a line-of-battle ship under full sail; how, among all the apparent confusion of ropes, and spars, and guns, and noises of duty, would he readily comprehend the machinery which is essential to the advance and guidance of the ship through the water? But put him first into a small boat, with a single sail or two, and let him try his hand at the helm, and sheet, or oar, and "principles" would become almost self-evident, and his visit *afterwards* to a ninety-gun ship would be under less disadvantages. It is just so with the young student of steam; give him a book in which the first half-dozen pages touch on the velocities of falling bodies, the law of expansions, deep researches into heat, &c. &c., and all appears to be confusion, or to him incomprehensible; but set him down to a work which, in plain language, systematically gives the main points of the subject, simply and properly illustrated, avoiding all unnecessary technicalities, and all theoretic

definitions which are not *immediately* called for; and, like the boy in the boat, he will have mastered the rough rudiments before too much is required of him.

We are rather a demonstrative people, and fond of similes; another illustration will not therefore be intrusive. The subject of teaching is treated herein with some novelty, as adapted to an especial purpose. The author therefore is anxious to justify, to those unacquainted with the success of his long labours, the course he has taken.

The mind of a well-educated engineer is a store of various kinds of knowledge, each of its peculiar character. A very large assemblage of subjects is indispensable; but the strength of the mind is manifested by the readiness with which each subject is made available to common professional purposes. The memory has stored up not merely facts, but theorems. All these have one tendency, which is to render the engineer able to perform a certain work under certain circumstances. Now let us compare the teaching of the early studies of such a man with the victualling of a ship. He is preparing for the voyage of life, and requires a long list of necessaries for his professional career, in the same manner that the ship needs a long list of supplies for the maintenance of her crew until she reaches her destination. What would the public think of the ship-owner who would say, at the outset of a voyage: "Well, really, bread is allowed to be the staff of life, and I cannot see what is gained by giving out beef and pork at the commencement of the voyage, for bread is an *indispensable part of human diet:* our constitutions should be first well accustomed to bread, and therefore bread ought to be, and shall be, the only early diet of the crew, say for the first half of the voyage—and it is so ordered"? But suppose such meagre and unvaried diet be found to impair the health of the crew—to render the appetite feeble and insensitive to a proper and natural

relish for food of any kind, and we have a parallel case to that of the teacher of engineers, who would prescribe bread-like theory *alone* as the early *indispensable part of the mental food* of the student. It is possibly the "dryness" of such bread which incapacitates one half our young aspirants for a course of study to which their minds would otherwise have been perhaps eminently adapted.

No plainer illustration of the author's desire can be given than the above. Repudiating the notion that theory *alone* is the most consistent early basis on which to teach useful knowledge, he would like in *his* teaching so to blend theory with practice, that while the man of theory should be ever attempting to *apply* his knowledge, the merely practical engineer should be ever craving for the "why and the wherefore."

RATIOS.

A ratio may be considered as a "relationship" or connection between, or comparison between, two numbers or quantities.

1. In all computations we must have certain data or parts given, in order to find (by comparison with some other part or quantity given) a certain result or consequence.

Therefore ratios form the basis of all calculation. The rule known as the "Rule of Three," is the combination of two ratios, and such is called a "proportion." This may be represented geometrically by the triangle, to which, therefore, we turn in so vast an amount of operations, and we depend upon trigonometry for the solution of questions of the highest scientific interest and importance.

It also is the basis of our ordinary daily household computations, even in the simplest possible form of the ques-

tion, viz. that of making shop purchases. As an example, I buy 6 lbs. of a substance at the rate of 2 lbs. for 5*s*. There are two ways of finding the cost of the 6 lbs. I may use the ordinary Rule of Three formula, and say: — As 2 lbs. : 6 lbs. :: 5*s*. : 15*s*. There is an impression that it would be absurd to say as 2 lbs. : 5*s*. :: 6 lbs. : 15*s*.; because, some ask, what comparison can we make between lbs. weight and shillings? But it should be understood that no one attempts to make such comparison,—we only compare abstract number with number. Let us split the above proportion into its two ratios, and see what we get,—

$$2 : 5 \text{ or } \tfrac{2}{5} \text{ and } 6 : 15 \text{ or } \tfrac{6}{15}$$

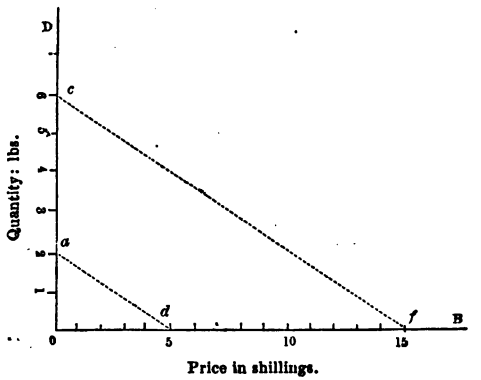
so that the ratios may be read $\frac{2}{5}$ is $\frac{6}{15}$, and there is nothing absurd in this. We may also form two sides of a triangle, letting the angle be as near a right angle as possible (for mere convenience), and dividing one side into equal parts to represent price, and the other to represent quantity, we proceed as follows: —

2. In the accompanying figure, suppose 0 B divided into equal parts, and 0 D into other equal parts (they may be from the same scale or not at pleasure); mark one side price, and the other quantity; lay off the quantity 2 lbs. from 0 to *a*, and make 0 *d* equal to 5*s*.; join *a d*; then laying off the other quantity given in the question, viz. 0 *c* = 6 lbs., draw from *c* a line parallel to *a d*, it will cut 0 B in *f*. Then 0 *f*, or 15*s*. is the answer.

3. We are indebted to Euclid for this solution, because he tells us that if a right line be drawn parallel to one of the sides of a triangle, it shall cut the other sides proportionally. (See Euclid, VI. 2.)

So that in the figure

Fig. 1.



the side $0a$: the side $0d$ :: $0c$: $0f$

or 2 : 5 :: 6 : 15

and in this case the 15 was found by the Rule of Three: the rule in which is — "Multiply the second and third terms together, and divide the product by the first, and the quotient will be the answer."

4. But we must know what this "Rule of Three" really is, and whence derived. If we turn to Euclid (VI. 16), we shall find that if four numbers are proportional, the product of the two means is *equal* to the product of the two extremes,—this is the origin of the term Equations.

EQUATIONS.

5. We can transpose the terms of a proportion by this theorem, so as the more readily to exhibit the value of any one term contained in it.

For in the proportions 2 : 5 :: 6 : 15

or 6 : 2 :: 15 : 5

or 15 : 5 :: 6 : 2

or 5 : 15 :: 2 : 6 &c.

it is seen that we can transpose, or invert, or change the terms of a proportional so as not to alter the values of either; for instance, such might have been written (4)

$$\left.\begin{array}{rl} & 2\times15 = 5\times\ 6 \\ \text{or} & 6\times\ 5 = 2\times15 \\ \text{or} & 15\times\ 2 = 6\times\ 5 \\ \text{or} & 5\times\ 6 = 2\times15\ \&c. \end{array}\right\} \text{in all } 30 = 30.$$

These again might have been placed so as to exhibit the precise value of each term in the proportion; for

$$2 = \frac{5\times6}{15}$$

$$6 = \frac{2\times15}{5}$$

$$15 = \frac{6\times5}{2}$$

$$5 = \frac{2\times15}{6}$$

and this was done in the first example by dividing both terms by 15: thus, if $2\times15 = 5\times6$, and I divide each term by 15, the work becomes $\frac{2\times15}{15} = \frac{5\times6}{15}$, or $2 = \frac{5\times6}{15}$; for in the left-hand side of the equation the fifteens cancel each other, so that $2 = \frac{5\times6}{15}$ remains as the equation showing the value of 2.

6. But we have supposed that the number 15 has been found from the other three data of the question, by the Rule of Three, and we want to exhibit the value of the unknown term before we find it—in other words we wish to show how it is to be found. We see already that $15 = \frac{6\times5}{2}$, but before the 15 was found we must have expressed fractionally the rule, or "formula," thus: —

$$\textit{The unknown number} = \frac{6 \times 5}{2}$$

multiplying and dividing would give us the 15 accordingly; but then this $15 = \frac{6 \times 5}{2}$ came from the proportion $15 : 5 :: 6 : 2$, and if the value of the 15 had not been known we must have said,

$$\textit{The unknown number} : 5 :: 6 : 2$$

But it is inconvenient to have to write such sentences as "the unknown number" in the midst of an equation: it is much easier to put some simple mark or symbol to designate such unknown term — any mark which is *readable*. Now, by common consent, mathematicians use one of the three last letters of the alphabet for this purpose; indeed, "x" usually means "the unknown number," or the "number we are seeking for;" therefore it would have been easier to write the above proportion thus,—

$$x : 5 :: 6 : 2$$

and its equation (4) would then be

$$x \times 2 = 5 \times 6$$

or, dividing each term by 2, it would become

$$\frac{x \times 2}{2} = \frac{5 \times 6}{2} \text{ or } x = \frac{5 \times 6}{2} \therefore x = 15$$

the answer required. Now, this use of a symbol is what we call "algebra," and the whole of the books on algebra are intended to teach us how to use such symbols as they occur in equations.

ALGEBRA.

7. It is a common complaint that generally works on the steam engine have their rules mostly in algebraic form —

but it might be a still stronger complaint that persons do not consider this "algebraic form" to be *nothing but vulgar fractions, with symbols instead of numbers.* Therefore, too much attention cannot be given to a thorough appreciation of fractions, both vulgar and decimal, to be immediately followed by at least elementary algebra — up to, say, simple and quadratic equations. There is no better work for a beginner than Colenso's, as it prepares for an extended advance into the study of mathematics. But if an officer has really little time to devote to algebra, &c., he will find herein the shortest possible abridgment of necessary work. The object of this book has been already declared to be the directing of studies under pressure for time, rather than the elaborate teaching; the meeting of the direct *necessities* of engineers and other officers so as to render their practical knowledge *available* to the public *while under* a course of proper study, and that the *meanwhile* should not be lost to themselves and their employers, at the same time that it lays a friendly plank or two along many a slippery and otherwise indistinct path.

8. Suppose, for example, we are considering the proper area of the steam passages from the boiler to the cylinder, and the pressure of steam in the cylinder after deduction due from the velocity of steam through contracted passages. Suppose we know as under:—

The force of steam in the boiler in inches of mercury . .	let us call	this	f
The velocity of the steam .	„	„	v
The force of the piston in inches	„	„	p
The area on the piston .	„	„	A
The velocity of the piston .	„	„	V
The area of the steam passage	„	„	a

Now, Tredgold's formula in this case is $v = \frac{p\,\mathrm{A}\,\mathrm{V}}{f\,a}$

The intelligent but imperfectly educated engineer, on examining this, will, in some disappointment, say, "It is not the velocity of the steam that I am seeking, but the area of the steam passage." Well, if he have read attentively (5) he will see that $v = \frac{p\,A\,V}{f\,a}$ means $v\,f\,a = p\,A\,V$; and by transposing, or, in this case, dividing both sides of the equation by $v\,f$, it becomes $\frac{p\,A\,V}{v\,f}$ or $a = \frac{p\,A\,V}{v\,f}$ which, in words, means thus:— the product of the quantities referred to as $p\,A\,V$, divided by the product, or sum produced by multiplication of the quantities referred to as $v\,f$, will give the area a required.

But as the object of this book is really to help the engineer in his work, it is better to incur the charge of prolixity than of inadequate brevity; therefore we will not leave this highly important question until it be even further illustrated. Let us attribute any values to v, p, A, V, f, and a; such, of course, as will satisfy the equation.

9. Say $v = 20$, $p = 6$, $A = 5$, $V = 4$, $f = 3$, and $a = 2$; then $v = \frac{p\,A\,V}{f\,a} = \frac{6 \times 5 \times 4}{3 \times 2} = \frac{120}{6} = 20 \therefore v = 20.$

And to find a we shall have from the equation given above, as already shown,

$$a = \frac{p\,A\,V}{f\,v} = \frac{6 \times 5 \times 4}{3 \times 20} = \frac{120}{60} = 2 \therefore a = 2;$$

and so on, until the values of the other single letters are obtained, and for practice the student will do well to find them.

Now this is the manner in which formulæ given in an algebraic appearance are to be treated.

From the facilities which vulgar fractions afford in simplifying the work to be done (through cancellings, transpositions, &c.), a knowledge of them should absolutely be at the "fingers' ends" of the engineer.

In order, therefore, to refresh the memories of those who have had but little recent practice in fractions, and have no time to regularly revise them in Colenso, the following will suffice,—given as worked examples rather than in the form of rules.

10. It is always understood that when a line is drawn under a number, it separates such number from its divisor, and merely saves the trouble of writing "*divided by;*" thus $\frac{5}{8}$ means 5 to be divided by 8; and again, using a complex fraction, $\frac{\frac{1}{2}}{\frac{2}{5}}$ means that $\frac{1}{2}$ is to be divided by $\frac{2}{5}$.

As a mixed number, $4\frac{3}{7} = \frac{31}{7}$ ∴ the improper fraction $\frac{31}{7} = 4\frac{3}{7}$.

As a compound fraction, $\frac{2}{9}$ of $\frac{6}{5}$ of $\frac{25}{3}$ of $\frac{5}{8}$ is resolved into factors, and like factors are cancelled thus,

$$\frac{2 \times (2 \times 3) \times (5 \times 5) \times 5}{(3 \times 3) \times 5 \times 3 \times (2 \times 2 \times 2)} = \frac{5 \times 5}{3 \times 3 \times 2} = \frac{25}{18}$$

Of course either of these numerators or denominators might have been mixed numbers or complex fractions; in all such cases they must be first reduced to simple fractions, having one numerator and one denominator.

11. Suppose a complex fraction like $\frac{8\frac{1}{2}}{15}$ had occurred; when reduced it would have become $\frac{\frac{17}{2}}{\frac{15}{1}}$, and standing thus as a complex fraction, the 15 has been made a fraction by putting a unit as a denominator, because the denominator always shows into how many parts the integer or whole

has been divided, so that $\frac{15}{1}$ merely signifies 15 *whole ones.*

N.B.—The $\frac{\frac{17}{2}}{\frac{15}{1}}$ must not be mistaken for a compounding of the ratios $\frac{17}{2}$ and $\frac{15}{1}$ because in reducing them we multiply 17 and 1, and 2 and 15; for this use of the products of the extremes and means (4), in forming the equivalent simple fraction $\frac{17}{30}$, arises not from any proportion implied in the two fractions, but simply from the circumstance that whether we multiply a numerator or divide a denominator we obtain the same result *in value.* For instance, if we multiply the fraction $\frac{5}{8}$ by 4, we find it $= \frac{20}{8}$ or $2\frac{1}{2}$, but if we divide the denominator of the fraction $\frac{5}{8}$ by 4, we have also $\frac{5}{2}$ or $2\frac{1}{2}$.

12. Multiplication and division of fractions are very easy; but in order to add or subtract them we must reduce all the fractions to the same denominator. For instance, if we add $\frac{1}{5}$ and $\frac{1}{8}$ and $\frac{1}{10}$ and $\frac{1}{16}$ together, we proceed thus: we multiply each term of each fraction by such a number as will bring them to the same denominator. In this case, as the 5 is a factor of the 10, and the 8 is a factor of the 16 (because $2 \times 5 = 10$ and $2 \times 8 = 16$), we may omit them in finding a least common multiple, and say $10 \times 16 = 160$, the new denominator; then $\frac{1}{5} + \frac{1}{8} + \frac{1}{10} + \frac{1}{16}$ will equal $\frac{32}{160} + \frac{20}{160} + \frac{16}{160} + \frac{10}{160} = \frac{78}{160} = \frac{39}{80}$. This might, however, have been shortened by calling $\frac{1}{5}$ by its equivalent $\frac{2}{10}$, and the $\frac{1}{8}$ by its equivalent value $\frac{2}{16}$, for then $\frac{2}{10} + \frac{1}{10} = \frac{3}{10}$, and $\frac{2}{16} + \frac{1}{16} = \frac{3}{16}$, and $\frac{3}{10} + \frac{3}{16} = \frac{48}{160} + \frac{30}{160} = \frac{78}{160}$, as before.

The following example, when worked out, will suffice for testing and refreshing the readers' memories as to fractions; the changes are arranged vertically for the purpose, and are better understood if worked in that order:—

Simplify $2\frac{1}{4} + 3\left(\frac{1}{2} \text{ of } \frac{1}{6}\right) \times \left(\frac{8\frac{1}{4}}{\frac{1}{3}} - \frac{\frac{3}{8} \text{ of } 4}{7}\right) + 6\frac{1\frac{1}{2}}{2}$

$= \frac{9}{4} + 3\left(\frac{1}{12}\right) \times \left(\frac{\frac{33}{4}}{\frac{1}{3}} - \frac{\frac{12}{8}}{\frac{7}{1}}\right) + 6\frac{\frac{3}{2}}{\frac{2}{1}}$

$= \frac{9}{4} + \frac{3}{12} \times \left(\frac{99}{4} - \frac{12}{56}\right) + 6\frac{3}{4}$

$= \underbrace{\left(\frac{27}{12} + \frac{3}{12}\right)}_{\frac{30}{12}} \times \underbrace{\left(\frac{1386}{56} - \frac{12}{56}\right)}_{\frac{1374}{56}} + \frac{27}{4}$

$= \frac{30}{12} \times \frac{1374}{56} + \frac{27}{4}$

$= \underbrace{\left(\frac{5}{2} \times \frac{687}{28}\right)}_{\frac{3435}{56}} + \frac{27}{4}$

$= \underbrace{\frac{3435}{56} + \frac{378}{56}}_{\frac{3813}{56} = 68\frac{5}{56}}$

TRIGONOMETRY.

13. In the author's work on Spheric Trigonometry, enough of the theory was given to show the principles of the science; the prefatory explanations there given apply equally to plane trigonometry, to which our attention is now called, and we shall therefore embody them in this book, marking such extracts with a *.

Whatever may be the rules used in solving questions in trigonometry, their proofs need not be explained until the student has learnt to use them with readiness.

In the author's work referred to†, it was not deemed necessary to introduce plane trigonometry in detail beyond its principles. In this, however, we shall give all that an engineer, or indeed navigator, can possibly require to enable them to solve all questions that can arise in their professions.

DEFINITION OF AN ANGLE.

***14.** Before proceeding it may be well to explain

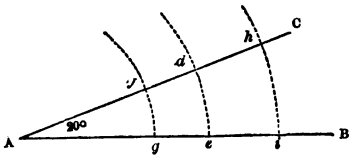
† "Calculation and Projection of the Sphere." Longman.

what is really meant by an angle: that such explanation is necessary cannot be denied. A work of this description, which is designed as a mere stepping-stone to study, must needs adopt assertions without proofs, for fear of alarming the timid who *desire* improvement, but who yet doubt their own powers. It may, however, be safely asserted, that since our proofs are deduced from the Books of Euclid mainly (93), a knowledge of his system of proving should be imparted at the earliest opportunity. In works upon navigation generally, more extracts from Euclid are given than the sea officer thinks necessary for his satisfactory working, and too few to satisfy his after-desire of research; while the Books of Euclid themselves are supposed to be too heavy an undertaking for any but a schoolboy having no other employment than study. These are delusions. A groundwork in mathematics well laid is a continual source of mental profit and *amusement*. There is no limit (but the powers of mortal intellects) to the structure which may be raised upon it. A very long acquaintance with the subject of teaching can only lead to a belief that whenever mathematical study is to any mind found to be repulsive, it may be suspected that the individual student *has not had its details sufficiently explained*. Thousands upon thousands can work an equation by logarithms who have but an indistinct notion of *what is really meant by an angle;* and it may cheer the student when he sees that we may go even to the "dreaded" Euclid to obtain a full and clear comprehension even of this trifle.

***15.** For instance, he says among his definitions (Book I. def. 8): "A plane angle is the inclination of two lines to one another in a plane, which meet together, but are not in the same direction." And (Book I. def. 9), "A plane rectilineal angle is the inclination of two right (or straight) lines to one another, which meet together, but

are not in the same right line; so that, in the following

Fig. 2.

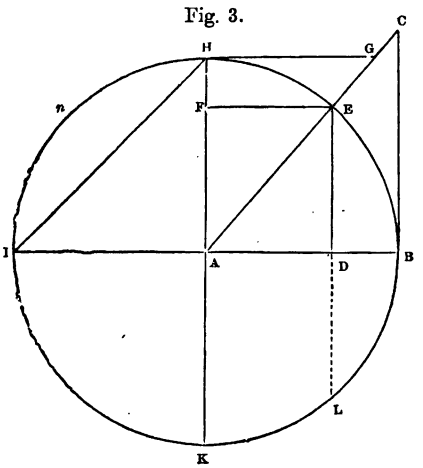


figure, the right line A B meets the right line A C at the point A, and the "*angle*" is the inclination of these two lines, as measured in degrees upon *any* circle drawn from A as a centre cutting these two lines. For instance, de, or fg, or hi, is each the measure of the "angle A" in degrees, 20 degrees meaning $\frac{20}{360}$ of *any* circle drawn round the point A (23).

TRIGONOMETRICAL CANON.

***16.** The following (Fig. 3) is called the trigonometri-

Fig. 3.



cal "canon" (a word merely signifying a collection of mathematical truths or formulæ), and from it are derived all the terms and rules used and practised in trigonometry.

In the above,

AB is called the radius of the circle (of course AK, AI, AH, and AE, are also radii).

BC is called a tangent (always of the opposite angle A), and is so called from Latin, *tangere*, to touch, because it only *touches* but does not cut the circle.

AC is called a secant (always of the angle between it and radius, therefore of ∠ (angle) A), so called from Latin, *secare*, to cut, because it cuts through the circle.

DE is called a sine, because it lies in the *hollow*, or bosom, of the curve EBL (Latin, *sinus*).

FE is called a cosine, or sine of an ∠ which is complementary to another, or required to make up 90°. Thus DE is the sine of the arc EB, and FE is the sine of arc EH, which is the complement of EB (for HE + EB = 90°). Therefore FE is called the cosine of EB, and is equal to AD, because FE and AD are drawn parallel, and ED is perpendicular to both.

GH is, in like manner, the tangent of HE, or cotangent of EB.

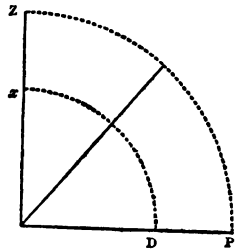
AG is, in like manner, the secant of HE, or cosecant of EB.

HI is called a line of chords, from its serving, as it were, to tie or confine the ends of the arc H*n*I.

DB is called a versed sine, and is the "height of the segment" EBLDE.

***17.** Every circle is supposed to be divided into 360 degrees (marked 360°). If, therefore, in the following, Fig. 4, PZ equals 90° or a quadrant, it is plain that if D*x* *also* equals 90°, the word "degree" refers to no measure of length, but merely signifies the 360th part of a circle whatever the size of that circle may be; and, therefore, a degree may be of *any* length. As, however, degrees enter into calculations, *some definite value* of them *must* evidently be necessary; and, consequently, geometers express the *value* of degrees by taking any two lines from those given in the trigonometrical canon, Fig. 3, and consider the *length* of one *as compared* with the *length* of another in the same triangle: so that we use the terms sine, tangent, secant (abbreviated sin, tan, sec, &c.), as referred generally to the radius of the circle, considering the length of radius to be 1 or 10, &c. (inches, feet, miles, leagues, &c., *at will*). This will be further illustrated.

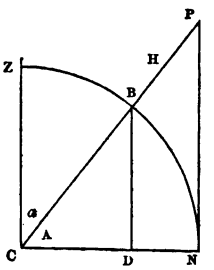
Fig. 4.



***18.** It has long been customary to call a *line*, as ED (Fig. 3), a sine, or as BC, a tangent; but such is only correct when the length of a radius of a circle is known or understood. It is generally useful to describe the sine, &c., as fractions, thus, $\frac{ED}{AE}$ (Fig. 3), as they express fairly the value referred to. Anticipating by a few pages the question of proportion, it may here be noted that a vulgar fraction is a "ratio" or proportion in itself, and is deduced from a triangle. Thus, when we speak of $\frac{1}{4}$ths of anything we refer to some magnitude which can only be appreciated by considering the fraction $\frac{1}{4}$ in relation to its integer or whole number "one." As this whole number

itself, expressed as a fraction, is $\frac{1}{4}$ths, $\frac{2}{3}$ths, $\frac{5}{6}$ths, &c.; and when, therefore, we speak of $\frac{3}{4}$ths, we express a "ratio," meaning as 3 is to 4 (or symbolically 3 : 4), so then we express the value of degrees by using ratios, and comparing them with the radius of the circle, which it is convenient to do by a number capable of decimal division, for obvious reasons (such as 1, 10, 100, &c.); and as at least one side of a plane triangle is always given, we are at liberty to compare this with the length of what is thus called the sine, tangent, secant, &c., of an angle, and hence the length of the arc itself.

Fig. 5.



***19.** For further example, in Fig. 5, let BD be what is commonly called the *sine* of the angle A. We describe its value by saying it is as the perpendicular is to radius, and write it thus: $\frac{\text{perp}}{\text{rad}}$, or, from the figure, $\frac{\text{BD}}{\text{CB}}$.

In like manner the other fundamental trigonometrical ratios are represented by fractions thus: —

$\frac{\text{perp}}{\text{rad}} = \frac{\text{PN}}{\text{CN}}$ is an expression for the tangent of $\angle$ A

$\frac{\text{hyp}}{\text{rad}} = \frac{\text{CP}}{\text{CN}}$,, secant of $\angle$ A

$\frac{\text{rad}}{\text{perp}} = \frac{\text{CN}}{\text{PN}}$,, cotan $\angle$ A

$\frac{\text{hyp}}{\text{rad}} = \frac{\text{CP}}{\text{PN}}$,, cosec $\angle$ A

$\frac{\text{rad}}{\text{hyp}} = \frac{\text{CN}}{\text{CP}}$,, cosin $\angle$ A

From the above it will be seen that certain ratios are *reciprocals*; for instance: —

$$\sin = \frac{PN}{CP} \quad \text{and cosec} = \frac{CP}{PN}$$
$$\tan = \frac{PN}{CN} \quad \text{and cotan} = \frac{CN}{PN}$$
$$\sec = \frac{CP}{CN} \quad \text{and cosin} = \frac{CN}{CP}$$

***20.** Therefore, in works on logarithms, when we want the secant of an angle we can find it by subtracting its cosin from 20 (the diameter of a circle whose radius is 10), and to find the log sin we subtract the log cosec from 20; and to find the log tan we subtract the log cotan from 20, &c. Other useful deductions may be made, such as to find the log tan: add 10 to the log sin, and from the sum subtract the log cos $\left(\text{or log tan} = \frac{\log \sin + 10}{\log \cos}\right)$ and to find the log cotan add 10 to the log cos, and from the sum subtract the log sin $\left(\text{or log cotan} = \frac{\log \cos + 10}{\log \sin}\right)$ &c.; so that the values of the six fundamental ratios may be expressed thus: —

$$\sin = \frac{1}{\text{cosec}} \qquad \text{cosin} = \frac{1}{\sec}$$
$$\tan = \frac{1}{\text{cot}} \qquad \text{cot} = \frac{1}{\tan}$$
$$\sec = \frac{1}{\text{cosin}} \qquad \text{cosec} = \frac{1}{\sin}$$

N.B.—The unit here meaning 1 diameter = 2 radii each of 10, or diameter = 20.

***21.** This, however, which forms the elementary base of

a proper knowledge of trigonometry, is not *absolutely essential* to the navigator, whose practical operations in plane trigonometry may be performed in total ignorance of principles, by dint of mere intelligence and skill from repetition; but in like manner does the blacksmith strike with the face of the hammer, and not with the handle, and would not probably perform his work more effectually if he were, previously to every blow, to calculate the force required to fashion his heated iron. But this must be remembered: a mathematical "blacksmith" would probably give *fewer blows*, because he would better know how to make each stroke tell, *from bringing the face of the hammer to bear on the iron in the best direction* with the greatest effect. In like manner the mathematical navigator will obtain his result in the shortest method.

It is beneath the dignity of a British sea officer to be content with mere knowledge of the use of formulæ. After reading this little book he may be safely advised to take up "Jeans's Trigonometry," or some such *small work* on the subject.

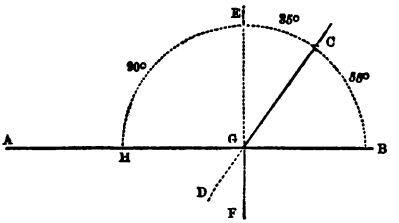
GEOMETRICAL THEOREMS.

***22.** A few of Euclid's theorems may here be introduced with advantage.

Book I. XIII.—The angles which one right line makes with another upon one side of it are either two right angles or are together equal to two right angles. Departing from the precise language of absolute and complete proof (for obvious reasons), we may say that the semicircle H E B contains 180 degrees. If E G be perpendicular to A B, the arcs H E and E B being equal will each contain 90°, but B C is less than 90°, being, say, 55°; then E C must be 35°, and E H being 90°, C H will be 125°. Now,

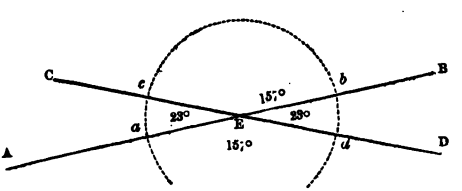
E C is called the *complement* of C B, and H C is called the *supplement* of C B.

Fig. 6.



***23.** Book I. XV. tells us that if two right lines cut one another the vertical or opposite angles shall be equal. Thus, the angles C E A and B E D are equal to each other, as are also C E B and A E D; the angle C E A means the angle at the point E formed by the two lines C E, E A (15). (We always put the letter *indicating the point* between the others.) Now, from the XIIIth Proposition, Book I., as above, it is evident that the two contiguous angles H G C and C G B equal 180 degrees; so in Fig. 7 $a\,c + c\,b$ equal 180 degrees, and $b\,d + d\,a$ form the other 180 degrees.

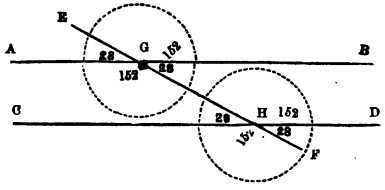
Fig. 7.



***24.** Euc. I. Prop. XXIX.—If a right line falls upon two parallel right lines, it makes the alternate angles equal to

one another, &c. &c.; so that, indeed, A G E, C H E, F H D, F G B, are equal to each other, being in this case about 28°, while E G B, E H D, F H C, and F G A are also equal, being about 152°,—the circles render this apparent.

Fig. 8.

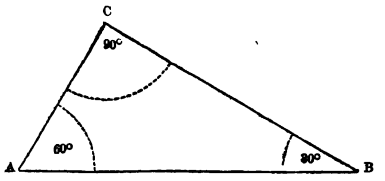


To satisfy this it needs only that A B and C D be precisely parallel.

***25.** Euc. I. Prop. XIX.—The greater angle of every triangle is subtended by the greater side, or has the greater side opposite to it.

The arcs of the circles drawn about each of the angular

Fig. 9.

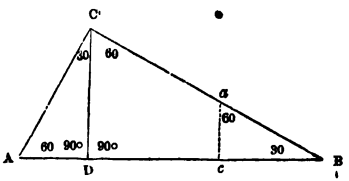


points with an equal radius show at once that, for instance, the small angle B 30° is opposite to the smaller side A C, &c.

***26.** Euc. VI. Prop. VIII. In a right-angled triangle if a perpendicular be drawn from the right angle to the base, the triangles on each side of it are similar to the whole triangle and to one another (that is, triangle A D C, triangle B D C, and triangle A C B are similar).

' Now, by *similar* triangles we mean triangles which have the three angles in the one equal to three angles respectively in the other; and although their opposite sides may be of different lengths, they are nevertheless proportional: thus, by the figure, if we lay off the distance C D at B *c*, and draw

Fig. 10.



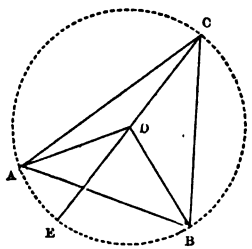
a c parallel to C D, we shall find the triangle B *c a* equal in its angles to C D A, and its sides proportional to triangle B C D, that is, B *c* will be to *a c* as B D is to D C (B *c* : *a c* :: B D : D C), &c. &c.; and here (by I. XXIX.), because B C falls across the two parallel lines *a c*, C D, the angles B *a c* and B C D are equal. (And this is the way in which one proposition of Euclid rests for proof upon others which precede. As an example, proof of the above proposition could only be made with *mathematical accuracy* by reference to 34 propositions of Book I., 10 of Book V., and 3 of Book VI.; in all 47 *propositions*, besides *definitions*, *axioms*, *and postulates*, repetitions, &c.)

***27.** Euc. III. Prop. XX.— The angle at the centre of a circle is double the angle at the circumference upon the same base, that is, upon the same part of the circumference.

In the triangle A D C, the side A D equals D C; therefore, as equal sides are opposite to equal angles (as deduced from Book I. Prop. XVIII.), the angle D A C equals D C A. But Prop. XXXII. Book I. implies that the angle

E D A equals D A C and D C A together; let A C B and A D B be two angles standing upon the same base A B; therefore, as A D E is the double of A C D, and by like

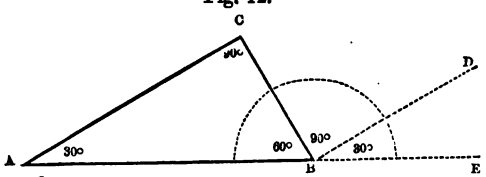
Fig. 11.



reasoning E D B would be the double of E C B, so must the whole angle A D B be the double of the angle A C B.

***28.** Euc. I. Prop. XXXII.—If any side of a triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of any triangle are equal to two right angles.

Fig. 12.



Reference to Prop XXIX. Book I. will show that the angle C A B will equal the angle D B E, because the right line C B falls on the two parallels A C and B D; also, that the angle A C B will equal C B D. Therefore the angle C B E, which is exterior to the triangle, and is made up of the two angles D B E and D B C, is equal to the two interior and *opposite* angles of the triangle (the angle A B C being the *adjacent* angle). And it is also evident that the

two "opposite" angles *together with* the "adjacent" angle which form the *interior* angles of the triangle are equal to the exterior and adjacent angles together, therefore (by I., XIII.) are equal to 180° or two right angles.

The above will be sufficient to give a general notion of the importance of Euclid.

PROJECTION OR CONSTRUCTION OF PLANE TRIANGLES.

29. No greater mistake can be made in the training of one who is to apply his knowledge of mathematics to practice, than in the neglecting of a proper early exercise in construction, or "projection," as it is called. It should especially precede the study of calculation of both plane and spherical triangles. It not only serves as a ready check upon calculation, and systematises our method of working, but in questions which frequently arise in practice it saves much ambiguity and imparts a steady confidence. It relieves calculation in general from the tedious monotony attendant on operations in which the intelligence and understanding are not appealed to, but where the memory alone is exercised. In the construction of a plane triangle an ordinary scale of equal parts and scale of chords are the only parts of the plane scale required.

30. Many students suffer considerable inconvenience at times from their reliance on the scale protractor, a very useful scale in the hands of the experienced, but it is plain that angles are measured by it with various degrees of accuracy; because, the scale being a long parallelogram, on it *small angles only* are upon a sufficiently large radius; larger angles, such as 70° to 90°, are measured on the smaller and *contracted* part of the scale, and the chord of 60°, as it is called (from a radius of a circle measuring as a chord 60° upon its arc), is sufficiently simple for even

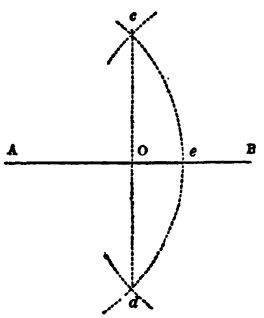
the beginner. The mode of using the chord of 60°, and a few necessary hints on projection, will be found among the following

GEOMETRICAL PROBLEMS.

31. PROB. I.—*To bisect a straight line,* AB, *that is to cut it into two equal parts.*

With any radius greater than half the length, and one foot of the compasses at A, sweep an arc as *ced*; with

Fig. 13.



the same radius and one foot on B sweep another arc across the first, intersecting at *c* and *d*. Join *cd*, and where this line cuts AB as at O will be a point equidistant from A and B.

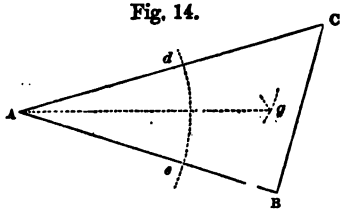
The same method applies to arcs of circles, and is used in drawing the points of the compass, &c., on a card.

32. PROB. II.—*To bisect a given angle* (*suppose the* ∠ A *in the triangle* ABC).

Take any radius less than the smaller of the two sides, and sweep an arc as *d e*. Take any other radius, and with one foot on *d* sweep an arc, shifting the foot to *e*

and crossing the last arc as at *g*. Join A*g*, and the line A*g* bisects the ∠ A.

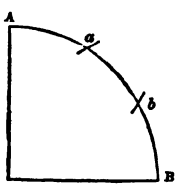
Fig. 14.



33. PROB. III.—*To divide a quadrant, or quarter of a circle, into three equal parts.*

Take the radius of the given circle on the compasses, and with one foot at one end of the quadrant, say at A, sweep it across the arc of the quadrant. Shift the foot to the other end of the quadrant, and do the same again. Then the quadrant AB, as in the figure, will be divided into three equal parts, as A*a*, *ab*, and *b*B, each of which will be 30°, or half the radius.

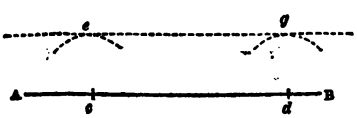
Fig. 15.



34. PROB. IV.—*To draw a line parallel to a given right line,* AB.

Assume any two points on the line AB, as *c* and *d*, and with the required distance at which the line is to be

Fig. 16.



drawn make the arcs *e* and *g*, and draw *eg* touching these arcs, and *eg* will be parallel to AB.

35. PROB. V.—*To erect a perpendicular from a point* a, *in a line*, BC.

Explanation is scarcely necessary, as the student will see the process in each case from the figures themselves.

When the point a *is near the end of the line.*

N.B.—*ab* is any distance assumed, and *cd* is made equal to *bc*.

Fig. 17.

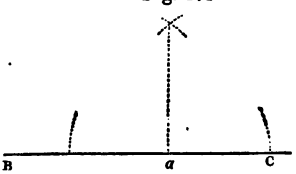
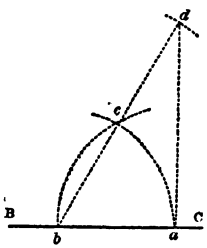
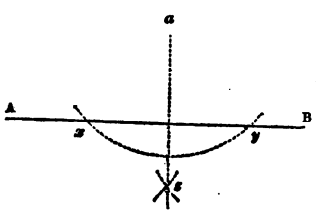


Fig. 18.



36. PROB. VI.—*To let fall a perpendicular from a point*, a, *on a given right line*, AB.

Fig. 19.

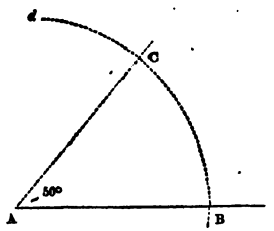


N.B.—*ax* is any assumed radius, as is also *xz* or *yz*, the two latter being equal.

37. PROB. VII.—*To make an angle equal to any given number of degrees, say* 50°.

Take 60° from any scale of chords, and from a point A in the line AB describe an arc as B*d*. On it, with the distance 50°, on the same scale of chords, lay it off from B to C, and the ∠CAB will equal 50°.

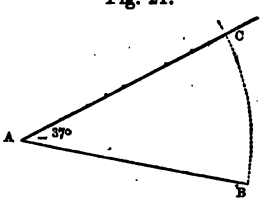
Fig. 20.



38. PROB. VIII.—*To measure an angle.*

With one foot of the compasses on A, and with 60° taken from any scale of chords, sweep the arc CB (it is

Fig. 21.



not necessary to draw the whole circle); then the distance CB laid off on the same scale of chords would measure say 37°.

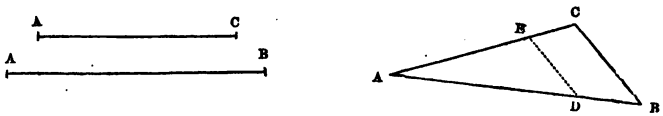
39. PROB. IX.—*To find a third proportional between two given lines,* AB *and* AC.

Place the two given lines, AB and AC, making any angle at A, and join BC. In AB take AD equal to

AC, and draw DE parallel to BC. Then shall AE be the third proportional to AB and AC.

Or, AB : AC :: AC : AE.

Fig. 22.

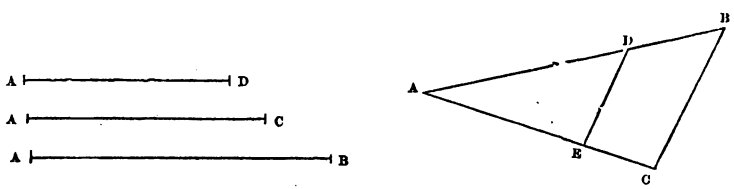


40. PROB. X.—*To find a fourth proportional to three given lines*, AB, AC, AD.

Place two of them, AB, AC, making any angle at A. Join BC. Place AD on AB, and draw DE parallel to BC, and AE will be the fourth proportional required.

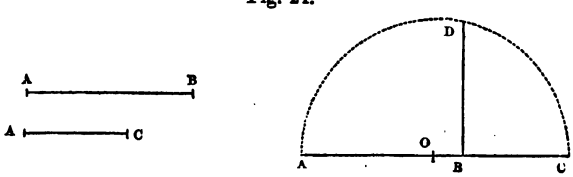
Or, AB : AC :: AD : AE.

Fig. 23.



41. PROB. XI.—*To find a mean proportional between two given lines*, AB *and* BC.

Fig. 24.



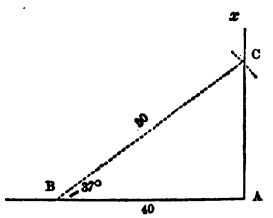
Join AB and BC in one straight line ABC, and bisect it in O. With centre O and radius OA describe a semicircle. Raise the perpendicular BD, and it will be the mean proportional required.

Or, AB : BD :: BD : BC.

42. Let us now take an example, and show the application of some of these to actual construction.

EXAMPLE.—Forty feet from the foot of a wall, stood a ladder 50 feet long, just resting on the top of it. Required the height of the wall, and the angle between the ladder and the ground.

Fig. 25.



Draw a base line to represent the ground, and draw another line perpendicular to it. (N.B.—It is customary to consider that to be the base line, or ground line, which lies parallel to the bottom of the paper or plan before us.) If we consider the perpendicular as Ax in the above figure to represent the wall, take 40 from a scale of equal parts, and lay it off on the ground line BA. Then take 50 from the same scale, and with one foot on B sweep an arc across the line Ax, and the point C will be the top of the wall, and BC, of course, the ladder. Measure AC off the same scale, and measure the $\angle$ B by Prob. IX. (39).

CALCULATION OF TRIANGLES.

43. Where sides only of a triangle are in question (as when using sides of similar triangles) calculation may be performed by common arithmetical proportion; but when angles are a part of the things given, we must have recourse to an artificial system, called logarithms.

All tables of logarithms have sufficient explanation for their use; consideration of principles may be deferred till farther on (71), and we proceed at once to the

RULE OF SINES.

44. In calculating parts of a triangle, whether plane or spherical, we shall often save much trouble if we first consider whether of the three things or parts given any two of them are a side and an opposite angle. When such is the case the "rule of sines," as it is called, founded on the fundamental theorem that "the sides of a triangle are in proportion to the sines of their opposite angles," is peculiarly simple.

To find a Side.

45. RULE.—As the sine of any given angle is to its opposite side, so is the sine of any other given angle to its opposite side.

To find an Angle.

46. RULE. — As any given side is to the sine of its opposite angle, so is any other given side to the sine of its opposite angle.

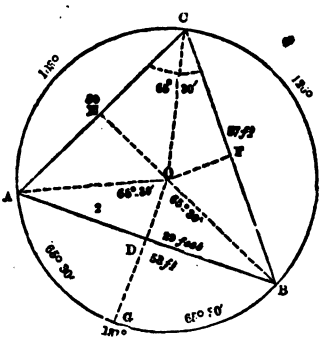
It is not, as already declared, the purpose of this book

to do more than give a plain but substantially practical introduction to the study of trigonometry, leaving the argumentative proofs of the various theorems to the few works on the subject which are already before the public.

47. But that which above has been called a "fundamental" rule deserves, in passing, a little attention, be it only to encourage the student towards further research; as in this he will see the simplicity of the study of geometry, when it is approached by a proper path.

In the following plane triangle ABC, bisect each side and erect perpendiculars; they will meet in O, the centre

Fig. 26.



(N.B.—Lengths and angles are marked in order that the student may easily verify by logarithms.)

of the circumscribing circle. Join OA, OB, and OC. Now, by reference to (27), we shall find that Euclid, in Book III. Prop. XX., proves that "*an angle at the centre of a circle is double the angle at the circumference upon the same base;*" therefore, in the above figure, the angle AOB is double the angle ACB; but by construction AD is the half of AB; similarly, the angle AOD is the half of the angle AOB; therefore, the angle AOD equals the angle ACB. Now, AB, the base of angles AOB and ACB, is a

chord of the arc ACB, and AD being half of AB (being by definition called a "sine") subtends the angle AOD or ACB.

Hence we find AD equals the sine of the angle AOD, equals the sine of the angle ACB.

By taking another base as AC, and another as CB, we shall, by the same method of demonstration, find that EC is equal to the sine of angle CBA, and also that FB is equal to the sine of angle BAC, and putting a for the side BC, and b for the side AC, and c for the side AB, we shall have the following equations: —

$$\tfrac{1}{2}a = \sin A \text{ (\textit{i.e.} sin of } \angle A)$$
$$\tfrac{1}{2}b = \sin B$$
$$\tfrac{1}{2}c = \sin C$$

And by combination: —

$$\tfrac{1}{2}a : \tfrac{1}{2}b :: \sin A : \sin B$$
Or, $$a : b :: \sin A : \sin B, \text{ \&c.}$$
Or, $$a : \sin A :: b : \sin B, \text{ \&c.}$$

Thus the sides are in proportion to the sines of their opposite angles.

A number of useful formulæ, which seem to wear so forbidding an aspect in ordinary works upon trigonometry, are really nothing more than easily obtained deductions from the above.

48. Triangles are either right-angled (have one angle equal 90°), or are oblique. The above rule of sines is a general rule for either, but when a side and an angle opposite are not among the parts given, we have recourse to other methods.

RIGHT-ANGLED TRIANGLES.

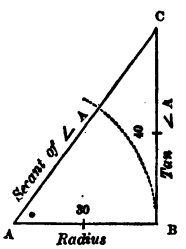
49. In these, if we consider one side as the radius of a circle, we shall give certain terms to the other sides as derived from the trigonometrical canon already explained (16).

It is generally best to make a *given* side radius.

EXAMPLE.—Suppose we have AB = 30 feet, and the side BC = 40 feet, required the side AC, and the angles A and C.

N.B.—In lettering a right-angled triangle, it is customary to place B at the right angle, and to call AC the hypothenuse.

Fig. 27.



50. It is better at first to *mark* the given sides or angles as in the figure. Now, if we make AB the radius of a circle, it becomes evident, on drawing a part of the circle, that BC will be a tangent of ∠ A, and AC will be the secant of ∠ A (16), and we write these names against them accordingly.

The following simple rules are then applicable.

To find an Angle.

51. RULE I.—As the side representing radius
Is to radius,
So is the other given side
To the name given to it by drawing the arc,
as tan, sin, sec, &c.

In applying this to the example, by the rule we say:—

As side AB . . . 30
Is to radius,
So is side BC. . . 40
To tan ∠ A.

Now, this being an analogy or proportion (5) (although the terms are placed vertically for the convenience of adding the logarithms), the product of the two extremes will equal the product of the two means, so that AB × tan ∠A = rad × BC; or, as ∠A is the part to be found, we place it alone on the left side of the equation (5), thus:—Tan $\angle A = \frac{\text{rad} \times BC}{AB}$, which means we have to add the logs of radius and BC together, because multiplication by logarithms is performed by addition (74), and subtract the log of AB, because division by logarithms is performed by subtracting. So that, indeed, the actual work of calculating this proportion will be as follows:—

As side A B 30′, .	1·477121	or ar co . . .	8·522879
Is to radius . . .	10·		10·
So is side B C 40′ .	1·602060		1·602060
	11·602060		20·124939
	1·477121	−10 I borrowed	10·
Tan ∠ A 53° 8′ . .	10·124939		10·124939

N. B.—We shall only herein work to the nearest minute of a degree,

52. It will be perceived that the result was obtained by adding the second and third terms, and subtracting the first, according to the ordinary "rule of three," as it is called in works on arithmetic. This might have been avoided by using a little artifice in the following manner.

ARITHMETICAL COMPLEMENT.

53. If we have to subtract 3 from 8, but yet for certain reasons wish to perform the operation ostensibly by addition, and we first subtract the 3 from 10, the remainder, 7, is called the arithmetical complement of 3. By adding the 8 and 7 together we get 15, and throwing off the 10 borrowed in subtraction, $15-10=5$. In the same manner, instead of subtracting the 1·477121 in the above logarithms from the sum 11·602060, if we had used the arithmetical complement (by subtracting the 1·477121 from 10), we might have performed the whole work by addition (as shown in the smaller type on the right-hand side of the logarithms above), and have moreover saved the risk of error in subtracting figures of different values. As it stands we did it thus:

$$\begin{array}{r} 10{\cdot}000000 \\ 1{\cdot}477121 \\ \hline 8{\cdot}522879 \end{array}$$

but a glance will show that we actually subtracted each figure, beginning at the *left*-hand side from 9, and the last from 10. As another example, the ar co (as usually written) of 3·824765 is seen at once to be (3 from 9 is 6, &c.) 6·175235.

Advantage will be taken of this trick in all our succeeding logarithmic calculations, from its abridging labour and diminishing risk of error, for it is much easier to subtract each figure from 9 than from various other promiscuously arranged numbers, and the habit is very easily acquired of reading off the ar co from the table at sight: and indeed some tables are arranged so as to have reciprocals placed next to each other, such as the sine and cosec (19) &c. We may then take off the log cosec for the ar co sine, or

vice versâ. If we notice, for instance, in the tables of logarithms, the sine of 36°, added to the cosec of 36° (or 9·769219 and 10·230781), equal 20 or twice radius (80). The log co arc sine of 36° is therefore 10·230781. But it must be remembered that this arithmetical complement applies only to such logarithms as are *subtractive*, being the first term of an analogy, or the logarithm of a number which if used *arithmetically* (5) would be a divisor.

54. Having thus found the ∠ A, the ∠ C is equal to 90° − ∠ A (or 90° − 53° 8′) or 36° 52′, because the internal angles of a triangle are equal to two right angles (Euclid I. XXXII.), and the ∠ B itself was 90°.

The side A C can now be found by the rule of sines, because an angle and opposite side are given (45).

To find Side A C (Fig. 27).

As sine of ∠ A 53° 8′ . . ar co	10·096892
Is to its opposite side 40 . .	1·602060
So is sine of 90°	10.
To its opposite side A C = 50 = .	21·698952

Dropping the 10 borrowed in taking the arithmetical complement, and also the radius 10 (53), leaves 1·698952 = 50.

To find a Side.

55. Rule II. — As the name of a given side (whether tan, sin, or sec, &c.)
Is to the given side,
So is the name of the side required (whether tan, sin, or sec, &c.)
To the side required.

All right-angled triangles can be worked by the above two rules and the rule of sines (45, 46).

OBLIQUE PLANE TRIANGLES.

56. Or those in which no angle equals 90°.

Of the six parts which belong to every triangle, viz. three sides and three angles, any three may be given, and of these there will be either,

CASE 1. A side and an angle opposite, with either another side or another angle among the parts given.

„ 2. Two sides and their included angle given.

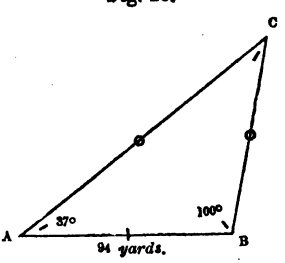
„ 3. Three sides given.

57. CASE I.

Examples under Case I. may be worked by the rule of sines entirely (45, 46).

EXAMPLE. — At one end of a garden walk I noticed a church steeple making a horizontal angle of 37° with it; at

Fig. 28.



the other end, the church made an angle of just 100° with it; the walk was 94 yards long: how far was the church steeple from me at each position?

Bearing in mind that the internal angles of a triangle equal 180° (54), if we add 37 and 100 and subtract the sum from 180°, the difference will be 43° = ∠C; therefore

$\angle$ C is virtually given, and as $\angle$ C is opposite the side A B, by the rule of sines (45), I say,

To find the side A C.

As sine $\angle$ C 43° . . . ar co	10·166217
Is to side A B 94′	1·973128
So is sine $\angle$ B 100° . . .	9·993351
To its opp. side A C 135,7	2·132696

Deducting radius and 10 borrowed in ar co (53).

To find side B C (45).

As sine $\angle$ C 43° . . . ar co	10·166217
Is to side A B 94′	1·973128
So is sine $\angle$ A 37°	9·779463
To side B C 82·95	1·918808

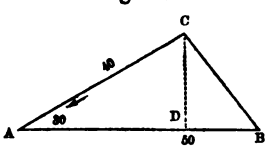
58. Case II.

Two sides and included angle given.

Examples under Case II. may be worked by rules already given, if we draw the figure so as to let fall a perpendicular upon the *longer* of the two given sides from its opposite angle.

Example.—Suppose two lines, as AB=50 inches, and AC=40 inches, make an angle at A=30°, required the length of CB?

Fig. 29.

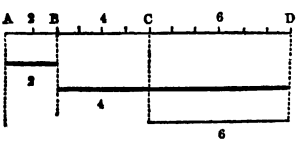


If, after having drawn the figure from the plane scale, we let fall a perpendicular (36) from C on AB, we

shall have in the $\triangle ACD$ the side AC and the $\angle$ A, besides the right angle at D, to find the rest by rule of sines (45, 46), because the $\angle$ at ADC, or radius, will be to the opposite side AC as the $\angle$ A is to its opposite side CD, or otherwise the $\angle ADC$, or radius, will be to its opposite side AC as the $\angle$ C $(90°-30°)$ 60° is to AD. And because AB is given in the question, and AD has been found, therefore DB is known (for AB−AD= DB). So that in the $\triangle CDB$ we now have CD and DB given, besides the right angle at D. Therefore we make one side radius, and proceed as in (55) to find CB; or, by Euc. I. XLVII., we say that as the square of DB added to the square of CD equals the square of CB, so CB will equal the square root of the sum of the squares of the sides CD and DB, or $CB = \sqrt{(CD^2+DB^2)}$.

59. But there is a simple formula for Case II. usually employed in practice, derived from the following consideration: —

Fig. 30.



If the line AD be divided into any number of equal parts (say twelve), and broken at any one of such parts, say at the second part, B, the length of each broken part will be equal to half the difference between those broken parts added to or subtracted from half the length of the whole part. Thus $AB=2=\frac{12}{2}-\frac{8}{2}=6-4$, and $BD=10=\frac{12}{2}+\frac{8}{2}=6+4$ (when $\frac{12}{2}$ equals the half length, and $\frac{8}{2}$ equals the half difference between 2 and 10 the given parts).

Hence we adopt a rule for the finding of the unknown angles of a triangle when one is given. It is this.

60. RULE.—As the sum of the two sides
Is to their difference,
So is the tan of half the sum of the unknown angles
To the tan of half the difference of the unknown angles.

And this tan half difference added to the half sum of two unknown angles gives the greater angle, and subtracted gives the less angle.

Taking the example already given (fig. 29):—

$AC + AD = 50 + 40 = 90$

$\angle A = 30°$ and $180° - 30° = 150°$, the amount of the unknown angles, the half which would be $75°$.

Then by the above rule $(A C + A D) : (A C - A D) :: \tan \frac{1}{2}(\angle B + \angle C) : \tan \frac{1}{2}(\angle B - \angle C)$;

Or, $$\tan \tfrac{1}{2}\text{diff} = \frac{AC - AD \times \tan \frac{1}{2}\text{sum}}{AC + AD}$$

By Logarithms:—

As $AC + AD = 90'$ . .	ar co	8·045757
Is to $AC - AD = 10'$ . . .		1·000000
So is tan $\frac{1}{2}$ sum of unknown angles $75°$		10·571948
To tan $\frac{1}{2}$ diff of the 2 angles $22° 31'$		9·617705

Then this $22° 31'$ added to $75°$ gives the greater $\angle = 97° 31'$, and subtracted gives $52° 29'$; and because greater sides are always opposite greater angles (Euc. I. XVIII.), $ACB = 97° 31'$, and $CBA = 52° 29'$. The side CB may then be found by rule of sines (45).

61. CASE III.

Three sides given:—

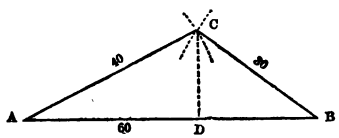
In this case we have no angle given, and our first object

is to find one. This is done by letting fall a perpendicular from the greatest angle upon the base, or greatest side, and finding the segments of the base thus cut by the perpendicular; having done which we are in a position to take advantage of the rule which we use in making one side radius (51), to find the angle opposite the segment, which will be the complement of the other angle at the base, and proceed thus: —

Given a side AB 60 feet
„ a side AC 40 „
„ a side BC 30 „

required the angles.

Fig. 31.



62. Having drawn the figure, and let fall the perpendicular at CD (36), the rule is: —

As the base or greatest side
Is to the sum of the two others,
So is the difference of those two sides
To the difference of the segments made by a perpendicular let fall from the greater angle upon the base.
Half this difference added to half the length of the base equals the length of the greater segment, and subtracted gives the less segment.

So that, from the above question, the equation would stand thus: —

diff of segts × base = sum of 2 sides × their diff

or, diff of segts × 60 = 70 × 10

$\therefore$ diff of segts $= \frac{70 \times 10}{60} = \frac{700}{60} = 11{\cdot}666$

Or, by logarithms:—

As base	60	ar co	8·221849
Is to	70	„	1·845098
So is	10	„	1·000000
			1·066947 = 11·666

Then $\frac{11{\cdot}66}{2} = 5{\cdot}83$ half diff

30·00 half base

35·83 = greater segt AD

24·17 = less segt DB

In $\triangle$ ACD, to find $\angle$ A (46):—

$$\text{Sin } \angle \text{ACD} = \frac{\text{rad} \times \text{AD}}{\text{AC}}$$

Or, by logarithms:—

As side AC	40	ar co	8·397940
Is to sin of	90°		10·
So is segt AD	35·83		1·554247

To sin $\angle$ ACD = $\bar{1}{\cdot}952187$ = 63° 36′ $\angle$ ACD

90

26 24 = $\angle$ A

The other angles are readily found by rule of sines (46).

63. *Enough has now been given to enable a beginner to compute any plane triangle whatever.* If triangles present themselves which appear to be too intricate for solution to one who has read thus far, it is most likely the difficulty arises from his not having properly attended

to *construction* by scale. Experience and habit give one facilities in working as in other subjects, but every *necessary* rule and every necessary explanation has been now given for plane triangles.

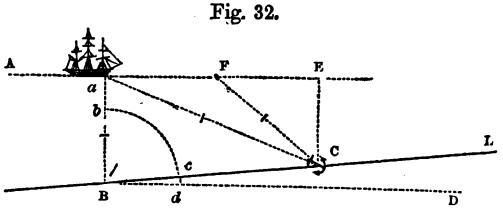
To practise the student in construction, let us take another example.

EXAMPLE.—I am riding in a tideway with 22 fathoms water under my bows. The bottom of the sea shoals ahead in the direction of the stream at an angle of 15°. I have 50 fathoms of cable out. What depth of water does my anchor lie in? And if I have 30 fathoms of buoy-rope, how far will my buoy watch ahead of me?

By construction:—

As the question refers to a vertical line (perpendicular of course to the horizon), we draw a line A E to represent

Fig. 32.



the water level, and assume a point as at *a* for the position of the ship's bows, from which we draw a perpendicular line, and make it (from any scale of equal parts) equal to 22 (fathoms); at the lower end of the line, which we will call B, draw B D parallel to A E (34), and draw B L so as to make an angle of 15° as D B L (37). Finding by the question that there are 50 fathoms of cable out, we take 50 (from the same scale) in the compasses, and sweep the distance across B L from the point *a*, and it will cut it at C. Now the vertical distance between C and E will be the depth of water the anchor lies in, and we draw C E accordingly. And again, the

question says that the buoy-rope is 30 fathoms long. Now, as the buoy-rope is made fast to the anchor, and will float in the direction of the stream, take 30 from the scale of equal parts, and sweep the distance across the water surface-line towards the ship's place from C, and F will be the buoy's place on the water.

Thus the whole question forms a vertical section of the water, and furnishes three distinct triangles.

The line E C is to be the answer, and, of course, we can *measure it at once* from the scale of equal parts, and get an answer *approximating in accuracy proportionate to the care bestowed on the construction*, and moderate care will give an answer abundantly accurate for all possible practical purposes. Now, such being the case, the student may be asked if it be consistent that so neat, and so easy, and so valuable an operation as construction should be neglected for calculation alone by formulæ which are at best difficult to recollect, and too frequently little understood? And when such happens to be the case, the student must abandon the subject altogether; whereas it may be safely affirmed that *what he can construct* he can calculate. Lest it be thought indecorous in the author to censure a popular practice, he would remind that an *accomplished engineer must be well acquainted with practical geometry and projection of various kinds —it is his chief work.*

64. The author is well acquainted in detail with the educational routine of the present age in these matters, and deplores the unsatisfactory and inadequate manner in which so fine a subject as trigonometry is, with some few exceptions, usually placed before students in this country. If he be in error, he has been unfortunate during a very long professional career in finding so very few who could readily apply trigonometry to practice— to general practice. If you talk of "departure" and "dif-

ference of latitude," the "course" and "distance" to a navigator, or of a parallelogram of forces to a superior mechanic, he at once attempts to apply, *not his knowledge* but his *recollection of formulæ* — making it a mere question of memory rather than of understanding. But take the very same data, and form with them a question of different aspect, and he would in most cases be at fault. Not very long since the author was requested to examine a class of young men in this subject, previous to their entering upon a higher routine of study, and only two out of nine could even use the scale of chords, although their memories had recently been well stored with formulæ; consequently only two out of the nine could measure or project an angle. And what is more, they had already passed a fair average examination in .... (but enough). This cannot be right. Would that the Universities (speaking with all respect for them) directed more attention to the subject in encouragement of "practical mathematics." The Dean of Ely has set good example. It is true that books furnish a whole archipelago of hints, but the surface of the system is dotted with so many seemingly detached islets, that one is apt to forget that these, which are but the more prominent portions, are firmly connected beneath the *not impenetrable* depths which surround them.

65. But to return to the question to which this not uncalled-for digression is due, occasions may occur in which scale and dividers are not at hand; or, further, extreme accuracy may be specially desirable; in such case the roughly sketched figure may be thus treated. Using letters, as in the above figure, we should first put a small mark across each given side (fig. 32), and an *o* against or upon the parts required. We should, moreover, fill in all the "derived" angles which do not appear in the question. For instance, the angle *a* B C is not given, but is *derived*

from $b\,c\,d$, which is a right angle or 90°, and $c\,d$ being 15° (per question) $90° - 15° = 75° = a\,B\,C$ (fig. 32).

Then in $\triangle\, a\,B\,C$ we can find $\angle a\,C\,B$ by rule of sines (46) thus: —

$$a\,C : \sin \angle a\,B\,C :: a\,B : \sin \angle a\,C\,B$$

Or, $$\text{Sin } \angle a\,C\,B = \frac{\sin \angle a\,B\,C \times a\,B}{a\,C}$$

Or, by logarithms: —

As side $a\,C$	50 .	ar co	8·301030
Is to sin $\angle a\,B\,C$	75°	. .	9·984944
So is side $a\,B$	22 .	. .	1·342423
To sin $\angle a\,C\,B$	.	. .	9·628397 = 25°9′

and $\angle a\,C\,B + \angle a\,B\,C = 75° + 25°\,9′ = 100°\,9′$: — and $180° - 100°\,9′$ leaves the $\angle B\,a\,C = 79°\,51′$; and as $\angle B\,a\,E$ is a right angle and $\angle B\,a\,C = 79°\,51′$, therefore $90° - 79°\,51′ = \angle E\,a\,C = 10°\,9′$.

Then in $\triangle\, a\,F\,C$ find $\angle a\,F\,C$

$$\therefore \sin \angle a\,F\,c = \frac{F\,a\,C \times a\,C}{F\,C}$$

Or, by logarithms: —

As side F C	30	ar co	8·522879
Is to sin $\angle F\,a\,C$	10° 9′	. .	9·246069
So is side $a\,C$	50	. .	1·698970
To sin $\angle a\,F\,C$		. .	9·467918 = 17° 5′
but the figure shows this $\angle$ to be obtuse			180 0
			$\angle a\,F\,C$ = 162 55

and $F\,a\,C = 10°\ 9′$

and $a\,F\,C = 162\ 55$

$173\ 4$

180

$\therefore \angle a\,C\,F = 6\ 56$

Then again in $\Delta\ a\,F\,C$ to find side $a\,F$ (45).

$$\text{Sin } \angle a\,F\,C : a\,C :: \text{Sin } \angle a\,C\,F : a\,F$$

$$\therefore a\,F = \frac{a\,C \times \sin \angle\ a\,C\,F}{\sin \angle\ a\,F\,C}$$

Or, by logarithms: —

As sin $\angle\ a\,F\,C$	$=162°\ 55'$	ar co	0·532082
Is to side $a\,C$	50		1·698970
So is sin $\angle\ a\,C\,F$	6° 56′		9·081759
To side $a\,F$		$=$	1·312811 $=$ 20·55

And then in $\Delta\ E\,a\,C$ find side $E\,C$.

$$\text{Sin } \angle a\,E\,C : a\,C :: \sin \angle E\,a\,C : E\,C$$

$$\therefore E\,C = \frac{a\,C \times \sin E\,a\,C}{\sin \angle\ a\,E\,C}$$

Or, by logarithms: —

As sin $\angle\ a\,E\,C$	90°	10·	
Is to side $a\,C$	50	1·698970	
So is sin $\angle\ E\,a\,C$	10° 9′	9·246069	
To side $E\,c$		0·945039	$=$ 8·811

Another example to illustrate "construction" will suffice.

Sailing along a coast, and noticing every appearance of a thick fog, I set a cape by compass E. by S. I then ran S. S. E. five miles, and found the cape bearing N. E. There was a dangerous shoal to the eastward, bearing from the cape E. N. E. ten miles. How should I have steered so as to pass four miles to the southward of it? (Fig. 33.)

The student may now fully rely on having had sufficient of Plane Trigonometry explained in the above to furnish him with a vast power of progress. He cannot now too

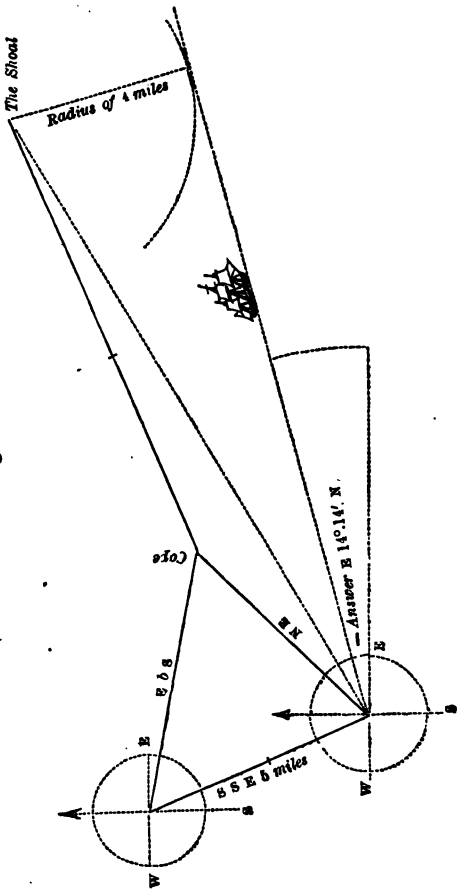

The Shoal
Radius of 4 miles
Answer N 14°.14′. W.
Core
N W
E b S
S S E 5 miles
N
E
S
W

Fig. 33.

soon take up such a theoretical work as Jeans's Trigonometry, and more especially should he provide himself with Jeans's admirable collection of questions, as published in a little work called "Problems in Astronomy, Surveying, and Navigation,"* wherein the work of practical trigonometry is illustrated by an extensive and useful variety of examples.

It is time now also to improve his acquaintance with Colenso's work on algebra. The equations to be found under various heads towards the end of this book on the study of steam, are, however, so arranged as to require no knowledge beyond that of vulgar and decimal fractions; but the study of algebra should, if time permit, precede the working of such problems or examples. Colenso's Algebra will be found very easy as a study, if every line and every word be thoroughly considered, as its admirable arrangement is such that each page forms a stepping-stone to those succeeding.

66. As many will have time to work beyond quadratics in algebra, some further remarks on the subject of equations may be acceptable, especially to those who have forgotten what they may have done on this subject years since. Too many are under the delusion that because a rule is understood after a few minutes' consideration, such is sufficient; and that one or two questions taken from the examples in each case are enough; but it ought to be remembered that the same data may come before a student in a variety of forms, so as to constitute a variety of beneficial exercises; and the self-taught will be well taught if they avoid impatience and hurry to begin something new. If any one thing more than another in early algebra is worth a student's labour, it is the use of "brackets." Readiness with these enables one to break up, or separate

* Longman and Co.

and simplify work to a great extent, more especially when we are using powers and extracting roots of numbers. The collecting of a number of like factors into one quantity will often give an entirely new aspect to an apparently complicated question, and Colenso, in particular, gives some highly useful examples. In no case, however, is the use of brackets more conspicuous than in what is called

THE BINOMIAL THEOREM,

67. Which is a ready means of raising any binomial, &c., without the tedious process of multiplication.

The following is the formula, and is in reality very simple, and easily remembered: the letter n being usually put to signify the exponent of the power to which the binomial is to be raised.

$$(x+a)^n = x^n + n x^{n-1} a + \frac{n(n-1)}{1.2} x^{n-2} a^2 + \frac{n(n-1)(n-2)}{1.2.3} x^{n-3} a^3 + \&c.$$

Suppose $n = 5$; then,

$$(x+a)^5 = x^5 + 5 x^4 a + \frac{5(5-1)}{1.2} x^{5-2} a^2 + \frac{5(5-1)(5-2)}{1.2.3} x^{5-3} a^3 + \&c.$$

or,

$$(x+a)^5 = x^5 + 5 x^4 a + 10 x^3 a^2 +$$

But as the exponent 5 is an *odd* number, the number of terms in the expansion will be *even:* they are always $n+1$, which in this case is 5 + 1, or 6, and as the two middle terms are always in such case alike, as regards the coefficients, and they repeat in reversed order to the last, we have as the fifth power of $(x + a)$,

$$x^5 + 5 x^4 a + 10 x^3 a^2 + 10 x^2 a^3 + 5 x a^4 + a^5$$

taking care to raise and diminish the exponents according to the rule evident in the first half of the expansion.

When the exponent is an even number, the number of terms in the expansion is always as above, $n + 1$, or an odd number, and there will of course be only one middle term.

Where the binomial is like $(x - a)^n$, we have only to remember that the sign of the second term in the series becomes negative, and that throughout the expansion the signs are alternate; so that the following formula will serve for either case: —

$$(x \pm a)^n = x^n \pm n x^{n-1} a + \frac{n(n-1)}{1.2} x^{n-2} a^2 \pm \frac{n(n-1)(n-2)}{1.2.3} x^{n-3} a^3 + \text{ \&c.}$$

and the powers of $x + a$ would therefore be as under,

$$x + a$$

2nd power or $(x + a)^2 = x^2 + 2xa + a^2$

3rd power or $(x + a)^3 = x^3 + 3x^2a + 3xa^2 + a^3$

4th power or $(x + a)^4 = x^4 + 4x^3a + 6x^2a^2 + 4xa^3 + a^4$

and it is a very profitable exercise to occasionally write out some of these expansions, especially if we make ourselves familiar with the use of brackets already mentioned, because we can easily by this means convert a trinomial, &c. into a binomial; —for example,

Required the expansion of $(a + b + c)^3$, by bracketing any two terms we get, for instance,

$$\{a + (b + c)\}^3 = a^3 + 3a^2(b + c) + 3a(b + c)^2 + (b + c)^3$$

and so with others.

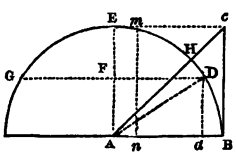
68. As the second power of a binomial occurs more frequently in practice, it is well to remember that the square of any such equals the sum of the squares of each of the terms + the double of their product.

A proper knowledge of the binomial theorem opens a wide door into the practice of higher algebra.

LOGARITHMS

69. May be used in calculation without the operator having the least idea of their nature; nor has it been found in practice hitherto that the mere mechanic needs more than the usual tables, with their attendant "directions for use." But as the navigator and marine surveyor, and more particularly the consistent engineer, will be desirous of something more satisfactory than mere tabular work, and will be sorely tempted by the nature of his employment to rise beyond the ordinary level of schoolboy attainments, somewhat of the principles on which logarithmic calculation is founded, and is called

Fig. 34.



for, will now be explained. In the above semicircle, if we lay off the length of the radius A B, as a chord, from E to D (A E being a perpendicular erected upon A B at A the centre), it will give the arc D B = 30°. If we draw D G parallel to A B, the line A F will be = D*d*, which is seen to be the sine of 30°, as also E A will be the sine of 90°, and *m n* the sine of (say) 80°. Now A F will be found to equal F E; hence the sine of 30° = ½ the sine of 90°. And again, if we bisect the arc E B (which is 90°) H B will be 45°, and *c* B will be the tangent of 45°; but the tangent of 90° is infinite; therefore *c* B, a finite right line, cannot be the half of *infinity*. These then seem to

set at defiance all our previous notions of proportion; for if we attempt, by common arithmetic, to solve such a question as sine 30° : 10 feet :: sine of 60° : x, we shall expect to get for answer 20 feet, while the truth would be 17·32, as easily seen by construction; and again, it is evident that the sine of 10° would be nearly half the sine of 20°, whereas the sine of 85° would differ very little from the sine of 90°.

70. To obviate the above inconvenience, an *artificial* system of numbers was sought for by mathematicians; and, happily, Lord Napier, about 250 years since, gave the world his system of Logarithms; and our astonishment is excited when we learn that this great discovery was made nearly half a century before what has since been called the "Logarithmic Series" was invented.

FORMATION OF LOGARITHMS.

71. The very word *Logarithm* is a stumbling-block to many; but it is easy to show that, although the ground-work is so very little known to many who use logarithms, a few hints put in familiar language will not merely gratify a laudable curiosity, but pleasantly assist in further investigation.

72. Lord Napier based his system of logarithms upon the following infinite series, in which it will be seen that values of fractions are systematically diminished by adding increasing multipliers to the denominators.

$$1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \frac{1}{1.2.3.4.5} + \&c.$$

Any one who understands vulgar fractions can resolve these into the following: —

$$1 + 1 + \tfrac{1}{2} + \tfrac{1}{6} + \tfrac{1}{24} + \tfrac{1}{120} + \&c.$$

Of course the series might be extended to great length and increased accuracy, but the above is enough for our purpose. Now, if we reduce the above fractions to a common denominator, we get

$$2 + \tfrac{60}{120} + \tfrac{20}{120} + \tfrac{5}{120} + \tfrac{1}{120}, \text{ &c.;}$$

and adding numerators we get $2\,\frac{86}{120}$, which, reduced to a decimal, becomes = 2·71666, an *approximate* base of the Napierian logarithms, which, if extended to further terms, and the division be made as usually shown in works on logarithms, becomes (as under) 2·7182818, &c., which, so far as it extends, is the *true* base of the Napierian logarithmic system; thus:—

$$1 + 1 + \frac{1}{1.2} = 2\tfrac{1}{2} = 2{\cdot}5$$

$$\frac{1}{1.2.3} = \frac{1}{6} = {\cdot}1666666666$$

$$\frac{1}{1.2.3.4} = \frac{1}{24} = {\cdot}0416666666$$

$$\frac{1}{1.2.3.4.5} = \frac{1}{120} = {\cdot}0083333333$$

$$\frac{1}{1.2.3.4.5.6} = \frac{1}{720} = {\cdot}0013888888$$

$$\frac{1}{1.2.3.4.5.6.7} = \frac{1}{5040} = {\cdot}0001984126$$

$$\frac{1}{1.2.3.4.5.6.7.8} = \frac{1}{40320} = {\cdot}0000248015$$

$$\frac{1}{1.2.3.4.5.6.7.8.9} = \frac{1}{362880} = {\cdot}0000027557$$

$$\frac{1}{1.2.3.4.5.6.7.8.9.10} = \frac{1}{3628800} = {\cdot}0000002755$$

$$2{\cdot}7182818 \quad \text{&c.}$$

Now, the Napierian system, arising, as above shown, from so simple an infinite series, and one which is so easily remembered, is made the basis of all other systems.

73. Any number may be taken as a base; let us at random take the number 3. Then, as logarithms are defined to be *a series of numbers in arithmetical progression, placed opposite to and corresponding with another series in geometrical progression, and so placed that* 0 *in the logarithmic stands opposite* 1 *in the geometric*—we can easily form a skeleton system based on the number 3, as under: —

Natural Numbers.		Geometrical.	Logarithms.
1	=	1	0·000000
3^1	=	3	1·000000
3^2	=	9 (3 × 3) . . .	2·000000
3^3	=	27 (3 × 3 × 3) . .	3·000000
3^4	=	81 (3 × 3 × 3 × 3) .	4·000000
3^5	=	243 &c. &c. . .	5·000000
3^6	=	729	6·000000
3^7	=	2187	7·000000
3^8	=	6561	8·000000
3^9	=	19683	9·000000
3^{10}	=	59049	10·000000

74. To prove that the above is really a table of logarithms, let us attempt calculation by it as we would by the table in common use: remembering the rules, that —

In logarithms, we *multiply* numbers by *adding* their logarithms, and we *divide* numbers by *subtracting* their logarithms. Suppose, for example, we desire to multiply 729 by 81. In the above table

the log of 729 is 6
log of 81 is 4
the sum 10 = log of the answer, 59049.

By arithmetic.

729
81
729
5832
59049

And again, to divide 2187 by 243:—

By arithmetic.	By logarithms.
243)2187(9	log of 2187 is 7
2187	log of 243 is 5
	the difference 2 = log of 9.

And further, to extract the cube root of 19683. This is done by *dividing the logarithm of the number by the index of the power*:—

log of 19683 = 9, and $\frac{9}{3}$ the index = 3, which is the log of 27;
or, as written, $\sqrt[3]{19683} = 27$.

And again, to raise the number 9 to the fourth power: multiply the log by the number of the index of the power; thus —

log of 9 is 2
index 4
8 = log of the number 6561;

or $9^4 = 6561$.

75. A table of the above description, as founded on the number 3, has, however, serious defects; the greatest is apparently the want of analogy between the number of figures in the whole number and the index of the logarithm, as will be shown immediately. The index or *characteristic* of the logarithm, is the integer, or whole number, and the decimal is called the *mantissa*. To facilitate calculation

by logarithms, Mr. Henry Briggs, a contemporary of Lord Napier, published at Cambridge, in 1615, a system having for its base the number 10, the root of our decimal scale of notation, in which the powers of the number 10 are shown by merely adding to unity as many cyphers as are "indicated" by what are therefore aptly called the indices of different powers, as we see in the following skeleton table to the base 10. Here, again, the logarithm is merely the *index of the power*, while it indicates absolutely the number of figures, *less one*, in the whole number to which it corresponds. This is the common system of logarithms.

76. The following table only gives the logarithms of numbers which are multiples or powers of 10, but we might, for instance, require to know the log of 270 — which would evidently lie between the log of 100 and the log of 1000; its index, however, would be 2 (because the number contains three figures), together with a decimal or mantissa, and we see in a more extended table it would be as represented by the log 2·23044.

Natural Numbers.		Logarithms.
10^0	= 1	0·000000 &c.
10^1	= 10	1·000000
10^2	= 100	2·000000
10^3	= 1000	3·000000
10^4	= 10000	4·000000
10^5	= 100000	5·000000
10^6	= 1000000	6·000000
10^7	= 10000000	7·000000
10^8	= 100000000	8·000000
10^9	= 1000000000	9·000000
10^{10}	= 10000000000	10·000000

77. It is important to add, that if the natural number

be a vulgar fraction, such as $\frac{5}{8}$, we may (because it means 5 divided by 8) subtract the log of the denominator from that of the numerator (increased by unity if necessary) thus—

$$\begin{array}{rl} \text{log of } 5 = & 0{\cdot}698970 \\ \text{,, } \quad 8 = & 0{\cdot}903090 \\ \hline & \bar{1}{\cdot}795880 = \text{dec. fraction } {\cdot}625. \end{array}$$

Proof $8\overline{)5000}$ $\quad {\cdot}625$

It is obvious, therefore, that it would have been as simple to have reduced the vulgar fraction to its decimal at once, and then to have taken its logarithm.

78. But we borrowed an unit in subtracting, therefore the resulting 9 was absolutely ·9 (decimal 9) or minus 1 (written $\bar{1}$). (A word here to those who are not "well up" in decimal arithmetic; be advised and lose not a day in "brushing up" a little. It is not, however, likely, that any one having sufficient interest in the subject to enable him to read this little book thus far, will do otherwise.) It will then be easily seen that it would, in the above example, have been better to borrow 10 than 1, writing the resulting index 9 as *minus* 9 ($-$ 9). Another example: multiply 100·6 by ·1006.

$$\begin{array}{rl} \text{log of } 100{\cdot}6 & = \quad 2{\cdot}002598 \\ \text{,, } \quad {\cdot}1006 & = -9{\cdot}002598 \\ \hline & \quad 1{\cdot}005196 \end{array}$$

Casting off the borrowed 10 it will be 1·005196, equal to the number 10·12; this is a more simple plan than writing the log of ·1006, as $\bar{1}{\cdot}002598$, and operating algebraically.

The following abstract will have its uses, and illustrate the above. (The number 1006 is taken at random: any number may be substituted.)

Natural Numbers.	Logarithms.
1006	3·002598
100·6	2·002598
10·06	1·002598
1·006	0·002598
·1006	—9·002598
·01006	—8·002598
·001006	—7·002598
&c.	

N.B.—All works on logarithms have rules attached, for taking out numbers, whether representing linear or angular quantities.

COMPUTATION OF LOGARITHMS.

79. We have seen (75) that only logarithms which have a certain base are conveniently applicable to practical purposes, and that the Napierian system is the most simply obtained from a *series* which gives its base 2·718281828, &c. This is commonly called the *natural* or hyperbolic system, and is written thus —

$(\text{Log}_{\varepsilon})$ 2·718281828, &c., while the decimal or Briggs's or the common system is written $\log_{10}$ (read, log to the base 10, or log to the base ε).

We use the Napierian system as a foundation of our common system in the following deduced formula: —

$$\left.\begin{matrix}\text{The common log}\\ \text{of any number}\end{matrix}\right\} = \frac{\text{Nap. log of number}}{\text{the Nap. log of 10}} = \frac{\text{Nap. log of number}}{2{\cdot}3025851}$$

$$= \frac{1}{2{\cdot}3025851} = {\cdot}43429488 \times \text{Nap. log of the number.}$$

Hence, to construct the common logarithm of any number, we use a number which may be called a *Napierian constant;* it is the double of the above, ·43429448 (which is the *modulus* of the common system), and equals

·86858896. It is here quite unnecessary to give the algebraic reasons why we use the following infinite series: it is enough for our purpose to say that in it we have a series, by which we may compute the logs of all natural numbers, and this without knowing the log of any previous number. It is this: —

$$\log P = 2M\left\{\frac{P-1}{P+1} + \frac{1}{3}\left(\frac{P-1}{P+1}\right)^3 + \frac{1}{5}\left(\frac{P-1}{P+1}\right)^5 + \frac{1}{7}\left(\frac{P-1}{P+1}\right)^7 + \&c.\right\}$$

Now, if we let P represent the number whose log we require, say the number 2, and M the modulus, ·43429448, the above will, in figures, be as under: —

$$\log 2 = 2(\cdot 43429448)\left\{\frac{2-1}{2+1} + \frac{1}{3}\left(\frac{2-1}{2+1}\right)^3 + \frac{1}{5}\left(\frac{2-1}{2+1}\right)^5 + \frac{1}{7}\left(\frac{2-1}{2+1}\right)^7 + \&c.\right\}$$

$$\log 2 = \cdot 86858896\left\{\frac{1}{3} + \frac{1}{3}\left(\frac{1}{3}\right)^3 + \frac{1}{5}\left(\frac{1}{3}\right)^5 + \frac{1}{7}\left(\frac{1}{3}\right)^7 + \&c.\right\}$$

(or, as reduced)

$$\log 2 = \cdot 86858896\left(\frac{1}{3} + \frac{1}{81} + \frac{1}{1215} + \frac{1}{15309} + \&c.\right)$$

(or, decimally)

$$\log 2 = \cdot 86858896(\cdot 333333 + \cdot 012347 + \cdot 000823 + \cdot 000065 + \&c.)$$

(or, after addition)

$$\log 2 = \cdot 86858896 \times \cdot 346568$$

(or, after multiplication)

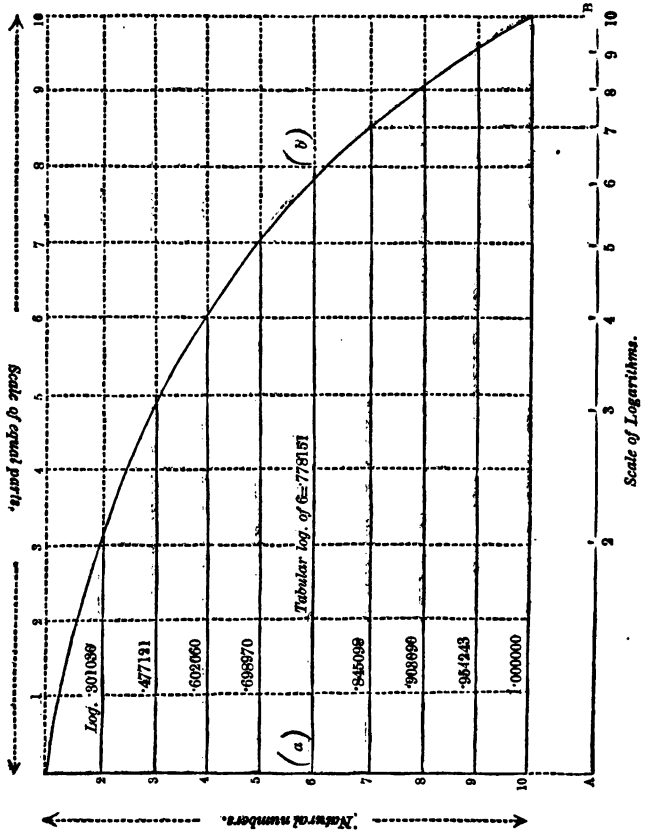
log 2 = ·301025 approximately, but if the series be extended
log 2 = ·3010300, as we find in our tables of common logarithms.

30. Perhaps no better notion of logarithms can be imparted than by giving the accompanying delineations of logarithmic curves.

Fig. A is drawn to show the connection between a scale of equal parts, and the logarithms of numbers.

For example, the logarithm of 6 is, by the tables, ·778151; it will therefore be seen that this number extends by scale of equal parts from a to b (equal to ·78 nearly), and so of the other numbers; the lengths of these

Fig. A.


Scale of equal parts.
Natural numbers.
Log. ·301030
·477121
·602060
·698970
(a)
Tabular log. of 6=·778151
(b)
·845098
·903090
·954243
1·000000
Scale of Logarithms.

lines are transferred to the scale AB, which forms the common scale of logarithms; and as to its properties we may see that if we add the length of (say) the log of 2 to the log of 4 the sum is equal in length to the log of 8 (·301030 + ·602060 = the log of 8, or ·903090), thus illustrating graphically that *addition* by logarithms is multiplication in common arithmetic. Various other properties may be seen in this scale by the intelligent student: as an example, if we desire by it to convert $\frac{3}{4}$ to a decimal, the distance between 3 and 4 laid back from 10 will reach ·75 the decimal required.

Figs. B and C illustrate in a similar manner the curves formed by laying off logarithmic distances of sines, tangents, secants, &c.: as an example (Fig. B), the log secant of 50° is by any table of logarithms = 10·191933. This number, in equal parts on the scale at the bottom of the figure, will therefore be 10·19, or it extends from *a* on the line of secants to *b*, through which, as on the points of the curve, the logarithmic curve is drawn. Logarithmic sines are also on the same figure. In Fig. C we have the same species of curve, only drawn through the extremities of lines measuring the logarithmic tangents, as taken out from the tables of logarithms, and laid off upon the scale of equal parts, the radius being 10.

Fig. D is interesting because it illustrates the actual meaning of certain formulæ (20), indispensable in trigonometrical analyses, such as, $\sin A = \frac{1}{\text{cosec } A}$ (supposing A to be 50°). By transformation (5) this becomes $\sin A \times \text{cosec } A = 1$; this means one *diameter*. For taking the log cosec 50° and log sin of 50° (or 10·115746 and 9·884254), the sum of the two is 20 = 1 diameter = 2 radii of 10 each (75); and in Fig. D, example 1, we see, accordingly, that the length of cosec 50° taken from Fig. B (being of course the sec of 40°), applied or added to sin 50°

Fig. B.

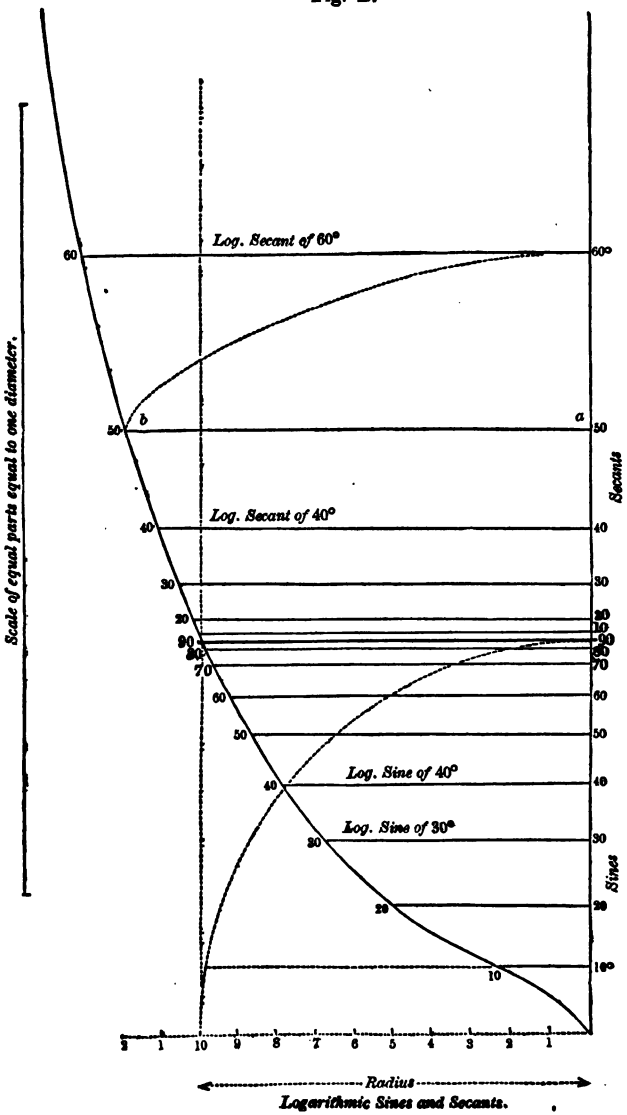

Scale of equal parts equal to one diameter.
Log. Secant of 60°
Log. Secant of 40°
Log. Sine of 40°
Log. Sine of 30°
Secants
Sines
Radius
Logarithmic Sines and Secants.

Fig. C.

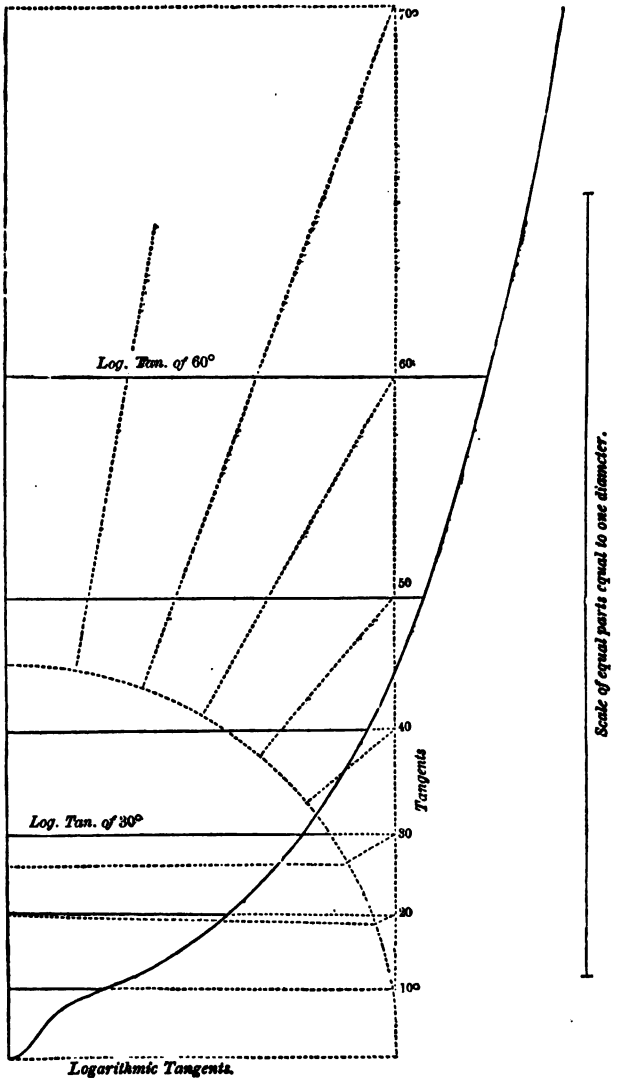
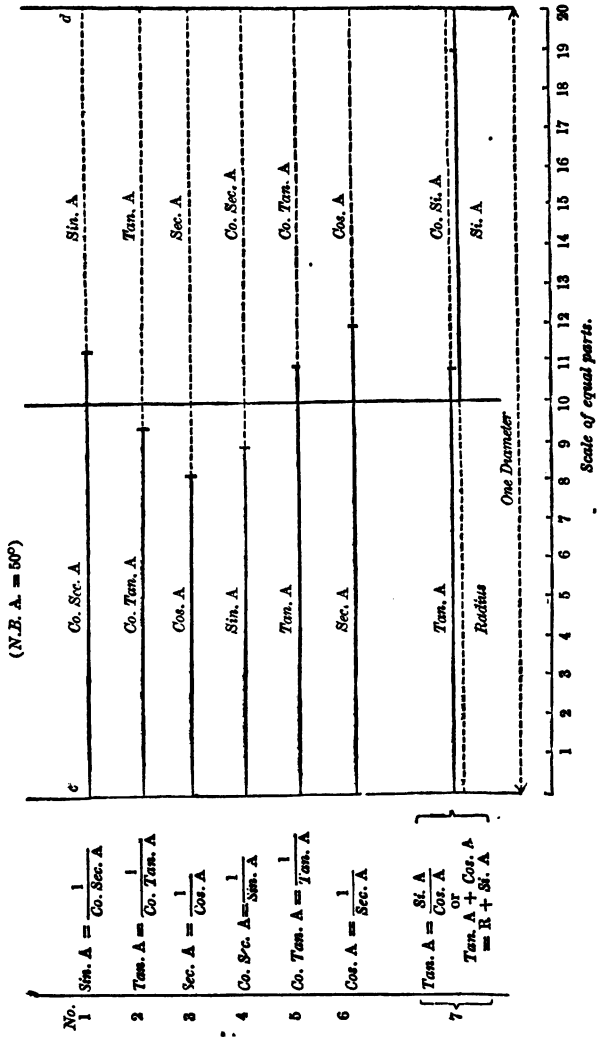

70°
Log. Tan. of 60°
60
50
40
Tangents
Log. Tan. of 30°
30
20
10°
Logarithmic Tangents.
Scale of equal parts equal to one diameter.

Fig. D.


(N.B. A. = 50°)
No.
1 Sin. A = 1 / Co. Sec. A
2 Tan. A = 1 / Co. Tan. A
3 Sec. A = 1 / Cos. A
4 Co. Sec. A = 1 / Sin. A
5 Co. Tan. A = 1 / Tan. A
6 Cos. A = 1 / Sec. A
7 Tan. A = Si. A / Cos. A or Tan. A + Cos. A = R + Si. A
c
Co. Sec. A
Co. Tan. A
Cos. A
Sin. A
Tan. A
Sec. A
Tan. A
Radius
Sin. A
Co. Sec. A
Tan. A
Co. Tan. A
Sec. A
Cos. A
Co. Si. A
Si. A
One Diameter
d
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Scale of equal parts.

from the same figure, B, make up the line *c d*, or a diameter in Fig. D. The same applies to the other equations therein given, and might be extended to any number.

Before entering upon a scientific consideration of the details of the engine, it is necessary to see that a proper knowledge of the method of calculation, &c., be secured. It is so wide a subject that mere description, without any appreciation of the principles on which calculations are made, would be injudicious; we shall therefore first take care that a proper acquaintance with mensuration be available.

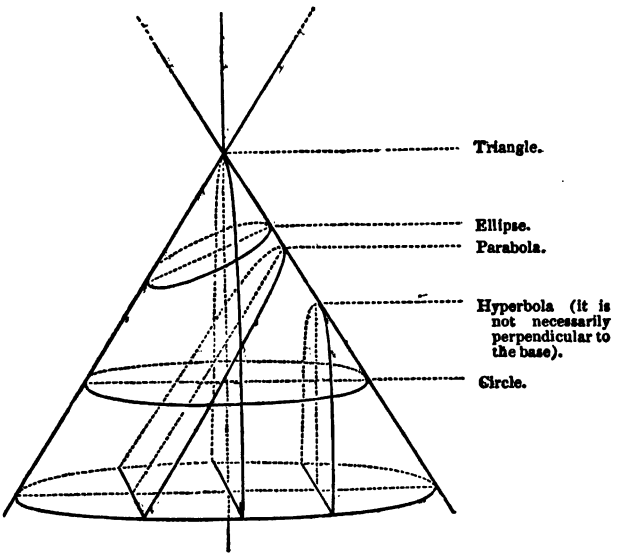
81. If we revolve a rectangular parallelogram upon one of its sides (fixed as an axis), we describe in space a cylinder. If we do the same with a right-angled triangle, we produce a conical figure,—a "right cone" as it is called. Now, all the work of calculation depends upon a correct knowledge of the various beautiful attributes of this figure; indeed, we propose to take it as our present study, and endeavour to impress on our minds the chief of its properties, as a constant resource whence we may often draw supplies, when mere unattached formulæ slip from the memory. The properties of the cone are developed by cutting the solid cone in various directions, and obtaining what are known as

CONIC SECTIONS,

and we are now entering upon a subject which has its limits only in the incapacity of the human intelligence. The loftiest exercise of the mind of man is in applying some of the principles of these conic sections to space. Of those who heard Sir John Herschel, in the Senate House at Cambridge, some years since, few will ever forget the towering majesty of the mind, in its power over matter, as

evinced by the illustrious astronomer, when he announced to his audience, that such was the power of mathematical study that astronomers were able to detect — even while he was speaking — the approach of some unknown body towards our solar system, or rather in it, from its already felt influence on, and disturbance of, the gravity of our planetary system. Shortly afterwards the planet Neptune was, for the first time, visible to mortal eyes. These triumphs of the mind rested for success solely on a rigid appreciation of the few figures now about to be ex-

Fig. 35.



plained herein. Nor is it to astronomy alone that these conic sections are so valuable: they apply to every art, to every branch of study, and will amply reward the engineer who devotes his mind to them. The following

will assist him over the elementary difficulties which some imagine to lie in the path of the study of them.

A right cone is defined to be a surface generated by the revolution of an *indefinite* straight line which passes through a given point making a given angle with a given straight line passing through that point.

82. If we suppose the sides of a cone to be produced to any length, we can only cut the cone by a plane in three directions, viz. (as shown in the figure), either as a

Parabola,
Ellipse, or
Hyperbola.

These are justly called physical curves, because they are described by the planetary bodies, by projectiles, &c., &c. We see in Fig. 35 five sections, but the circle is merely a limit of the ellipse, as the triangle is of the hyperbola.

In using figures called conic sections, we should distinctly understand that whatever curve we have under consideration, such curve is supposed to be traced by a particle in motion.

THE CIRCLE

83. Enters so largely into practical applications that it merits first illustration.

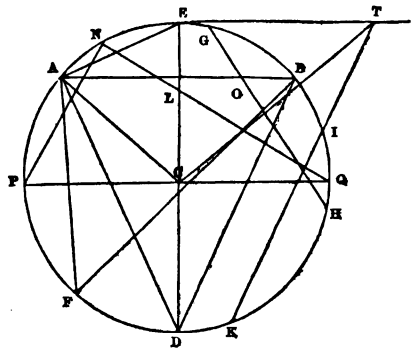
A circle is a continuous line so drawn that all its parts lie at an equal distance from a point called its centre.

The following are its principal properties: —

If C be the centre
ECD is a diameter
AB is a chord of the arc AEB
AE is a chord of half the arc AEB

ET is a tangent to the point E, or of the arc EB
AC is a radius (also PC, EC, BC, &c.)
AEBCA a sector
AEBA a segment.

Fig. 36.



ANGLES.

∠ACB is an angle at the centre.
∠ADB is an angle at the circumference.
∠ACB is double ∠ADB. (Euclid III. XX.)
∠AFB is equal to ∠ADB. (Euclid III. XXI.)
∠PNQ, ∠PEQ, ∠PAQ, &c. are equal to right angles. (Euc. III. XXI.)

RECTANGLES.

EL × LD = AL × LB, written EL.LD = AL.LB, and read "rectangle EL.LD," &c. (Euclid III. XXXV.)

$(ET)^2$ or $ET.ET = KT.TI$. (Euc. III. XXXVI.)

$AO.OB = GO.OH$, or $AO = \frac{GO \times OH}{OB}$ (5), &c.

TRIANGLES.

DAE is similar to AEL because AL is a perpendicular let fall upon DE.

$$\therefore \angle AEL = \angle DAL$$
$$\angle LAE = \angle ADL$$
$$\therefore EA : AD :: EL : AL \quad . \quad . \quad . \quad 1.$$
$$DL : AL :: AD : AE \quad . \quad . \quad . \quad 2.$$
$$DA : DL :: AE : AL \quad . \quad . \quad . \quad 3.$$

N.B. It will be very useful practice if the student take out the values of the several parts as under: —

From 1. $EA \times AL = AD \times EL$ (5)

or, $EA = \frac{AD \times EL}{AL}$

or, $AL = \frac{AD \times EL}{EA}$, &c.

THE PARABOLA.

A parabola is such that two straight lines drawn from any point in its curve, the one towards a point called the *focus*, and the other perpendicular to a straight line given in position, called the *directrix*, these straight lines will always be equal to each other.

TO CONSTRUCT A PARABOLA.

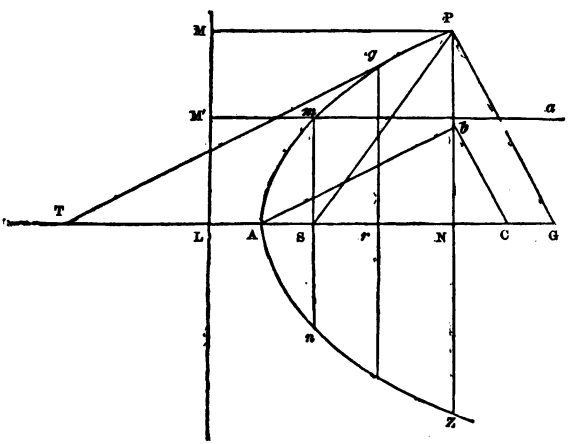
84. Given — an ordinate to the axis;
the abscissa to that ordinate.

Draw two lines at right angles to each other, and make PN = half the given ordinate; let NA = the abscissa.

Bisect the ordinate PN as at *b*, join A *b*; draw *b* C at right angles to A *b*, then NC will be the focal distance, which lay off at AS, and draw *m* S *n* parallel to PN.

Make AL = AS, and draw ML at right angles to LA.

Fig. 37.



Then draw any number of ordinates across AG, as *g r*, &c.; and, for example, take L *r*, and from the focal point S sweep that distance across *r g*, and the point *g* will be a point in the parabola. Do the same with any other ordinates, and connect the points of the curve so found, and the curve will be a parabola of which,—

ML is the directrix.
A is the apex.
S is the focus.
AG is the axis.
PN an ordinate.
AN an abscissa to ordinate PN.
mSn the latus rectum.
M′ma a diameter.
PT a tangent to point P.
NT a subtangent.
PG a normal to the tangent.
NG a subnormal to the tangent.

It is worthy of remark, that a most perfect parabola is frequently to be seen in the section of the thin end of a hen's egg. A few mornings before this was written, the author chopped with his knife upon the end of an egg-shell at table, and finding the resistance of the shell to be very great, requiring a very firm blow with the edge of the knife to cut it, he examined the curve — it proved to be a true parabola, thus illustrating the strength of the "arch."

SOME PROPERTIES OF THE PARABOLA.

The latus rectum equals four times the distance of the focus from the vertex (or $m\,n = 4$ AS).

The tangent at any point bisects the angle between the focal distance and diameter through the point (or $\angle$ MPT = SPT).

The subtangent equals twice the abscissa (or TN = 2 AN).

The subnormal equals half the latus rectum (or NG = $m\,s$, and NG = 2 NC).

The rectangle under the latus rectum and the abscissa is equal to the square of a semi ordinate of the axis, or $(PN)^2 = m n . AN$.

AREAS OF THE PARABOLA.

85. When the abscissa and its ordinate are given.

RULE.—Multiply the base by the height, and take two thirds of the product for the area, or $\frac{2}{3}(PZ \times AN) = \text{area}$.

The area of a parabolic frustum is obtained by taking half the difference between the two areas of the two abscissæ to the two ordinates; or, when the two double ordinates and their distances are given,

RULE.—Add the product of the two double ordinates to the sum of their squares, divide the result by the sum of the two double ordinates; the quotient multiplied by two thirds the altitude of the frustum will be the area.

N.B. If we remove the portion of a parabola which lies between the apex and an ordinate, the other part is called a frustum.

THE ELLIPSE.

86. An ellipse may be described as a figure in which a point (as P) in traversing the outline has the sum of its distances from two fixed points (the foci) always the same.

N.B. No part of an ellipse can be drawn with the compasses.

TO DRAW AN ELLIPSE.

Given — the axis major AM.
„ the axis minor BE.

Draw two lines at right angles to each other crossing at

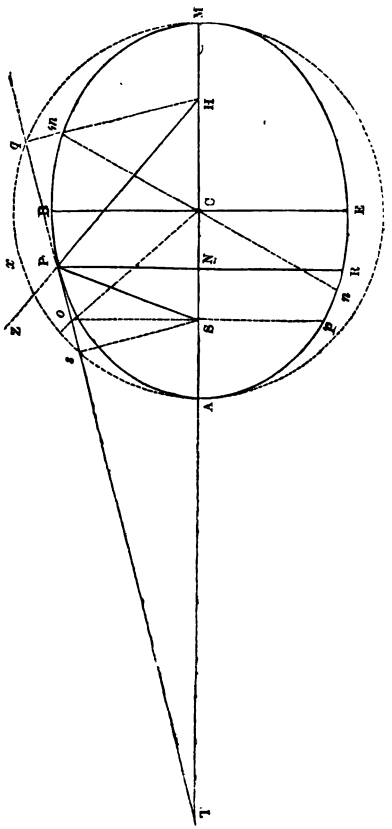


Fig. 38.

C as a centre: lay off half the axis major from C to A and C to M, and half the axis minor from C to B and from C to E. With BE as a radius and one foot in M, cut AC in S; make MH equal to AS. Then S and H are foci to the ellipse, and HC or SC is its *eccentricity*.

PNR is an ordinate to abscissa AN.
o s p is a latus rectum.
m n is a diameter.
A and M are vertices.
PT is a tangent to point P.

SOME PROPERTIES OF THE ELLIPSE.

The sum of the focal distances of any point is equal to the axis major, as PS + PH = AM.

The tangent at any point makes equal angles with the focal distances (or $\angle$ ZPT = $\angle$ SPT).

The perpendiculars from the foci on the tangent intersect the tangent in the circumference of a circle of which the axis major is a diameter, as *q* H and *s* S intersect the circle A *s q* M.

The rectangle under the perpendiculars from the foci on the tangent is equal to the square of the semi axis major;

$$\text{or, } q\,\text{H}.\,\text{S}\,s = (\text{AC})^2.$$

The sum of the squares of semi diameters is constant;

$$\text{or, } (m\,c)^2 + (\text{C}\,o)^2 = (\text{AC})^2 + (\text{BC})^2.$$

The product of a semi ordinate and semi axis major equals the product of semi axis minor and the corresponding semi ordinate of the circumscribing circle;

$$\text{or, NP} \cdot \text{N}x :: \text{CB} : \text{CA}.$$

The latus rectum is a third proportional to the axes major and minor;

or, AM : BE :: BE : *op.*

The properties of the ellipse are so varied that we can only take such as are commonly useful.

TO FIND THE AREA OF AN ELLIPSE.

The product of the axes major and minor multiplied by ·7854 gives the area.

The area of a portion of an ellipse cut off by an ordinate is to the area of a corresponding segment of the circumscribing circle as the semi axis minor is to the semi axis major;

or, area ANP : area AN*x* :: CB : CA.

THE HYPERBOLA.

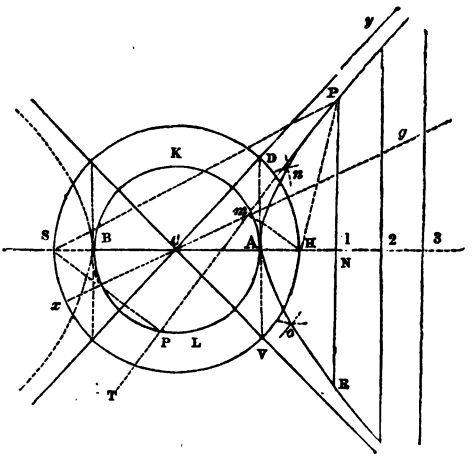
The hyperbola is a plane curve, such that if from any point in the curve there be drawn two straight lines to any two fixed given points, the lengths of such two straight lines shall always have the same difference.

TO CONSTRUCT A HYPERBOLA.

87. In Fig. 39 let C be the centre of the hyperbola in the given axis major AB: at A raise a perpendicular, and on it make AD equal to half the axis major (for a right hyperbola); join CD, and with it as a radius describe a circle; the points H and S (in the axis major produced) will be the foci. Assume points in the same line, as 1, 2, 3, &c., and with the distance (say) A 1 from

focus H, and B 1 from focus S, make intersections as at n and o; and n, o, will be points in the hyperbola.

Fig. 39.



Proceed in the same manner with A 2, &c., and when sufficient points are obtained draw a curve through them.

Thus, S and H are foci.
g C x, a diameter.
A B axis major.
A and B are vertices.
A N and B N are abscissæ to P N.
P N R is an ordinate.
K L the axis minor.
C y is an asymptote.
S C or C H the excentricity.
D v is the conjugate.
T P is a tangent to the point P.

SOME PROPERTIES OF THE HYPERBOLA.

The difference of the focal distances from any point is equal to the axis major (or SP − PH = BA) (or Sn − nH = BA).

The tangent to any point makes equal angles with the focal distances (or ∠ SPT = ∠ HPT).

Perpendiculars from the foci upon the tangent intersect the tangent upon the circumference of a circle, having the axis major as its diameter (or S P is perpendicular to TP, and cuts it on circle BKAL) (also Hm is the same).

The rectangle under the perpendiculars from the foci on the tangent is equal to the square of the semi axis major (or m H, SP = (BC)^2).

TO FIND THE AREA OF THE HYPERBOLA.

To the product of the axis major and abscissa add $\frac{5}{7}$ths the square of the abscissa, and multiply the square root of this sum by 21; to this product add 4 times the square root of the product of the axis major and abscissa, and divide the last by 75 times the axis major; the quotient is the area *very nearly*.

MENSURATION OF SUPERFICIES.

88. To find areas.

For the square or any rectangular figure. — Multiply length by breadth.

N.B. Mind that the dimensions are of the same denomination, such as both feet or both inches, &c.; and if the product be in inches, divide by 144 for feet, and if feet by 9 for square yards, &c.

For the triangle. — Half the product of its base and height.

For the regular polygon or figure of four or more sides having all the sides equal. — Let fall a perpendicular from the centre upon one of its sides, multiply this by the side itself and by the number of sides in the polygon, and half the product is the area of the polygon.

For the trapezoid, a four-sided figure having only two sides parallel. — Half the sum of the two parallel sides multiplied by the perpendicular distance between them is the area of the trapezoid.

For the trapezium, a four-sided figure having no sides parallel.—Better divided into two triangles by a diagonal. Find the sum of the areas of the two triangles.

89. *For the circle.*—The area of a circle whose diameter is 1 is ·7854; therefore, when the diameter of a circle is given to find the area,—square the diameter and multiply by ·7854.

The area may also be found by multiplying the circumference by the diameter and dividing by 4;

$$\text{(or, area of circle} = \text{di} \times \cdot 7854) \text{ or area} = \frac{\text{cir} \times \text{di}}{4}$$

For the sector of a circle or any part of a circle bounded by an arc and two radii. — Half radius multiplied by the length of the arc equals the area.

N.B. To find the length of an arc. From eight times the chord of half the arc subtract the chord of the whole arc, and take one third the remainder for the length of the arc *nearly*.

For the segment or any part of a circle bounded by an arc and its chord. — Find the area of the sector and the triangle; the difference is the area of the segment.

90. *For finding the areas of circles,* &c.—It is necessary to remember the numbers ·7854 and 3·1416; the first is one fourth the latter, and we may use it thus for areas:—

As the area of a square whose side is 1 : the area of its inscribed circle ·7854 :: the area of any other square : the area of its inscribed circle.

and again —

As a circle whose diameter is 1 : its circumference 3·1416 :: the diameter of any other circle : to its circumference.

N.B. An inscribed square is half the area of the circumscribed square.

The circular ring. — Find the area according to the greater diameter, and subtract the area of the inner circle from it.

The ellipse. — The product of the axes major and minor multiplied by ·7854 gives the area.

The parabola. — Multiply the base by the height, and take two thirds the product for the area.

The hyperbola. — To the product of the axis major and abscissa add five sevenths the square of the abscissa, and multiply the square root of this sum by 21; to this product add 4 times the square root of the product of the axis minor and abscissa, and divide this last by 75 times the axis major; the quotient is the area *very nearly*.

MENSURATION OF SOLIDS.

91. *For the cube or parallelopipedon.*—Multiply the length by the breadth and that product by the depth, and the result will be the solidity.

Cylinders or prisms.—Find the area of the base, and multiply by the height.

The cone or pyramid. — Multiply the area of the base by the height, and one third the product will be the solid content.

The frustum of a cone. — Add the squares of the two diameters of the ends to their product; this sum multiplied

by one third the perpendicular height and again by ·7854 will give the solidity of the frustum.

The sphere or ball. — Cube the diameter and multiply by ·5236 $\left(=\frac{3{\cdot}1416}{6}\right)$; the product will be the solidity.

The segment of a sphere. — To three times the square of the radius of its base add the square of its height; then multiply the sum by the height, and the product by ·5236.

The cylindrical ring. — Add the inner diameter to the thickness of the ring, multiply the sum by the square of the thickness and the product by $2{\cdot}4674\left(=\frac{3{\cdot}1416^2}{4}\right)$, and it will give the solidity.

SUPERFICIAL AREAS OF SOLID BODIES.

The sphere or ball. — Multiply the diameter by the circumference, or multiply 3·1416 by the square of the diameter.

The convex surface of a cylinder.—Multiply the circumference by the length of the cylinder for the area.

The convex surface of a right cone. — Multiply the circumference of the base by the slant height or side of the cone, and take half the product for the area.

The convex surface of a cylindrical ring.—Multiply the extreme diameter by the thickness of the ring, and the product by 9·8696 $(=(3{\cdot}1416)^2)$, and it will give the area.

N.B. It is not intended that the above should supplant the various rules and course of study given in works on mensuration: such only are given, above, as it may be convenient to the junior officer to know, before he has time for regular study.

SIMPSON'S RULE FOR AREAS.

92. When more than ordinary accuracy is required in finding the area of a space lying between a curve and a straight line.

Divide the base of figure (suppose it to be the following) into any even number of equal parts, say eight (the more we take the greater accuracy we attain), by drawing lines at right angles to the base; this will give an odd number of points of division. The measurements of the lengths of these perpendiculars, or *ordinates*, must be taken from a scale of equal parts and used as in the following, which is called Simpson's Rule: —

Let S = the sum of the lengths of the first and last ordinates.

E = the sum of the lengths of all the other *even* ordinates.

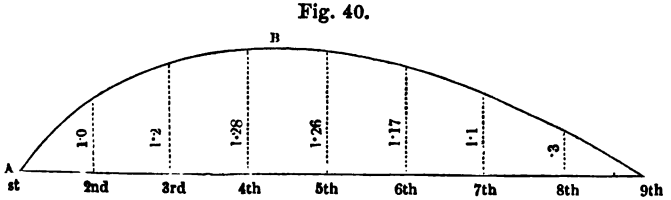
O = the sum of the remaining or *odd* ordinates.

d = the common distance between the ordinates.

$$\text{Then the area} = \frac{(S + 4E + 2O) \times d}{3}$$

For example: Given the following figure, ABC, required its area: —

Fig. 40.



S = 0

4 E = 4 (1·0 + 1·28 + 1·17 + ·3) = 4 (3·75) = 15

2 O = 2 (1·2 + 1·26 + 1·1) = 2 (3·56) = 7·12

d = 1

$$\text{Then } \frac{(S+4E+2O)\times d}{3} = \frac{0+15+7\cdot12}{3} = \frac{22\cdot12}{3} = 7\cdot37$$

This rule is much used by shipbuilders in calculating displacements, water-lines, &c., and is on the general assumption that the curve is parabolic; it may also be applied to the "working off" the indicator diagram. (*Indicator*, 148).

EUCLID.

93. Before commencing in detail the calculations involved in the study of steam and the marine engine, a few remarks on Euclid's geometry will assist in the better comprehension of that most important work — a work not to be neglected by any in whose practice mathematical science enters largely. One hears of "logical proof" and "mathematical demonstration," and each requires to be properly estimated, in order to arrive at correct conclusions whenever anything is being weighed in the mind, or, as we term it, is "under consideration." Now consideration is but another term for argument (in its ordinary application), and if, as we are told, every argument has two points, viz. that which is proved, and that by which proof is obtained, it is evident that the mode of such proof should be in accordance with some principles of reasoning, as proof must involve thought, and science is the result of thought. It is science which has essentially for its object the determination of "what is" and "what is not." But the study of logic is a wide one, and leads so deeply into *metaphysical* labyrinths, whither the general student may well hesitate to follow, that one turns with gladness to another species of logic (for such is mathematical demonstration) in order to seek its aid in the determining of "what is" or "what is not" in *physical* science. And geometry, as arranged by the ancients, admits of considerably wide application in those sciences which relate to number, space, and position. In short, Euclid's geometry, with all

its imperfections, seems adapted to supply to the ordinary mind what logic gives to the refined and deeply contemplative.

There is something in the human mind which seems congenial to the study of geometry. It is no right ground of argument to say that many persons have little capacity for mathematical study. The author believes that many young people, who are said to be stupid and dull over "figures," are for the most part furnished with fair average powers of mind. Many such individuals are to be found who would drive four-in-hand, or conduct a steam engine, manage a boat, &c., with dexterity and *judgment*. Will it be said, then, that such minds are incapacitated for any other pursuit? Those who have minds of sufficient *force* and strength to enable them to perform even the average work of active life, are surely able to apply their force and powers to other subjects than those to which chance (if such a thing there be) may have led them. After all, what is man's occupation as taken individually? Is it a path invariably chosen with due regard to mental qualifications, or is it, in *most* cases, one entered upon as a matter of will, taste, training, or accident? Admitting that "training" has had anything to do with one's avocation, it may be said that one could have been also as easily trained for geometry.

The multitude find least difficulty in comprehending what is most frequently presented to them — familiarity with an educational subject induces an interest in it. The mind may, it is true, be said to have a "bias" towards, for instance, the acquirement of languages; but remember, it is quite probable that such bias is totally independent of capacity and intellect, for circumstances might quite as easily have "biassed" the mind towards some other pursuit.

People, as an accepted rule, admire what is beautiful in

Nature and Art, and have a certain impression of pleasure at beholding the "wonderful." It is neither symmetry, nor colour, nor form, that alone excites our admiration. It may be questioned even if the pleasurable impressions we receive are those of sight merely; it is the impression that what we see makes upon the mind, with which we have to deal in speculating upon its intelligence or capacities. Nor need we limit the sphere for such impressions to a wonderful natural production, or a wonderful production of art, or a wonderful scientific development; each is capable of moving our minds to agreeable appreciation; and why may not geometry and its developments and curious applications claim equal power to interest when circumstances favour the quiet inquiry into such matters? Besides, are we not told that the mind is strengthened by judicious exercise, and therefore the first step is to *promote the habit of such exercise* by placing our subjects before students in an agreeable form, or as much so as a consistent mental discipline permits. These remarks are called for, inasmuch as those for whom this book is expressly intended, having mostly passed the usual age of school study, will be liable to be influenced by recollections of attempts at Euclid which in their experience have given rather a disrelish for theoretical geometry. Such persons may be assured that want of success has arisen solely from the inadequate manner in which propositions have been studied. *It is impossible for any one to understand Euclid's propositions, and not to like them.*

It is the same with most other sciences — take chemistry, for instance. Take a number of totally illiterate persons into your laboratory, and, as an example, put a few drops of sulphuric acid to some sugar, and exhibit the black charcoal so easily obtained from it. Not one of the spectators will depart without satisfaction and *desire to see more*, and the intensity of this desire will be in proportion

to the pains taken in explaining the nature of the change in the sugar. And so it is particularly with geometry. As the mind begins to comprehend the properties of certain figures in common use, and their ready application to our various needs, an interest arises, which, if encouraged, will strengthen as the student proceeds.

And it is just this encouragement which this little book is intended to offer. Let the student commence Euclid, and attack his first proposition. If he have a symbolical edition, so much the better for his purpose. If it prove too much for him in its peculiar mode of argument, it will be so because the first line of the enunciation speaks of what seems to want definition. Reference to preface or previous matter seems at first to offer no help in such cases; indeed the preface of one excellent author uses the following as almost the only *advice:* — "Each syllogism should be rendered clearly distinct from the other, in order that," &c. &c. Now this, so far from aiding, presents a new obstacle, because the student was not expecting to find "syllogisms" in his path. But further, in tracing up the meaning of the word that first seemed to need explanation, he will find among the "definitions" the true meaning of equilateral triangles, but in doing so he encounters what absolutely appals him. He is told that a point is invisible (or what is the same thing, that it has no part, no magnitude), that a line is therefore invisible (having no breadth); these are at least fair inferences to be drawn by one who thinks he is entering upon a rigid argumentative course of demonstration. And further, he is told that an angle is the inclination of two right lines, &c. (equally, of course, invisible). Nor does this state of uncertainty receive a check until perchance the eye reads the words in the postulates, "Let it be granted." To an intelligent student the nature of mathematical demonstration now begins to dawn upon him. He sees that in Postulate I.

he is allowed to make marks, and *call them* lines; that he may *call* certain figures circles; that he has to rest his arguments on definitions, postulates, and axioms. Turning again to Book I. Prop. I., Euclid will now seem to be bargaining thus with his readers: "You wish to understand geometry, and *prove* its propositions? I will teach you how to do it, if you will allow me to attach certain meanings to the words I use: for instance, when I speak of an obtuse angle, let it be understood, as in my 'definitions,' that it means an angle greater than a right angle. Let me, moreover, assume that I can do certain things which I cannot prove possible to be done, such I call *postulates;* and let me have the benefit of using (as unproved but self-evident) facts which are called axioms, and I will in the first proposition prove that I *can* draw an equilateral triangle on a given finite right line."

If the student keep this in view, and refer to the definitions, &c., in aid of his proofs as necessary, the study of Euclid becomes not merely pleasant but enticing. The so much abused *pons asinorum*, and many other propositions, may be more readily understood by drawing each triangular part *separately*.

Book II. is amusing as showing the direct connection between geometry and arithmetic, for all its propositions can be worked out by simple calculation.

SECTION II.

ELEMENTARY MATHEMATICS APPLIED TO MECHANICS.

A FEW of the leading rules, which are prominently adapted to the wants of the engineer, will be given in connection with leading principles.

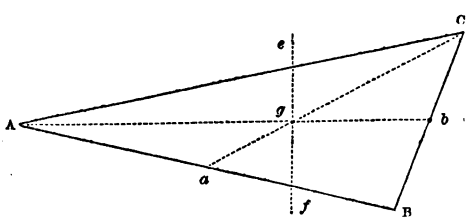
In considering

GRAVITY,

94. The effect of the whole power upon the mass of the body is supposed to be concentrated at one point, the position of which varies according to the shape of the mass itself; but there are easy rules for determining the position of this point. Every material mass has its centre of gravity.

The centre of gravity of a plane triangle may be found as follows. In the triangle A B C bisect any two sides,

Fig. 41.



as A B and B C, as at a and b; join a C and b A, and their point of intersection, g, will be the centre of gravity of the triangle, and $g\ b$ will equal one-third of A b, or $a\ g$ will

equal one-third of *a* C, in all cases; and the position of the centre is such, that if the triangle were cut out from the page and pinned (by a fine pin) through the intersection at *g* to the side of an upright post, it would remain in whatever position it was placed: in other words, it would be in a state of equilibrium, as regards the earth's attraction upon the different parts of the mass, inasmuch as no part of the figure being turned towards the earth's centre would displace the centre of gravity; but this would not be the case if the pin were placed elsewhere. We can from this method find the centre of gravity of any irregular flat body; for, supposing the figure A B C to have been such irregular body, on suspending it by a pin at A, and drawing a vertical or plumb line towards *b*, and suspending it next from C, and drawing the plumb line C *a*, we should get the same point of intersection, *g*, as the centre of gravity, as already given. We may from this question of gravity of the triangle easily illustrate the law under which we calculate the powers of the lever, and show the interesting connection between forces, gravity, velocities, &c. For if we consider the line A *b* in the triangle, we shall see that the mass on the right of the line *e f* (which is here purposely drawn perpendicular to A *b*) must counterpoise the mass on the left side; and as $g\,b = \frac{1}{3}$ of A *b*, so, supposing the imaginary lever itself to have no weight, 1 lb. hung at the end of such lever at A would counterpoise 2 lbs. hung at the point *b*, for it is a principle of the lever that the power (P) at A, multiplied by A *g* (supposing *g* to be fulcrum), would equal the weight W at *b* multiplied by the length of *g b*, or $\mathrm{P} \times \mathrm{A}\,g = \mathrm{W} \times g\,b$; and so of the line C *a*. The lever, therefore, is only a question of "gravity," and all calculations thereupon are deduced from certain compensations which we call *equilibrium*.

THE LEVER.

The first kind have the fulcrum between the power and the weight.

The second kind have the weight between the power and the fulcrum.

The third kind have the power between the weight and the fulcrum, or, in other words:—

The first kind have the fulcrum in the middle.

The second kind have the weight in the middle.

The third kind have the power in the middle.

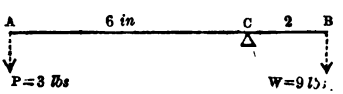
N.B. The weight is often called the *resistance*.

95. The fundamental theorem for the calculation of the lever is, "that to produce equilibrium the moment of power equals the 'moment' of weight." The word "moment" being an expression for an amount which is the product of the mass and *velocity*; the "velocities" being in such cases represented by the lengths of arcs, described by the ends of the arms of the lever (as moving on its fulcrum) in the same period of time. Levers, theoretically considered, are rigid bars without weight in the beam, pressing on a point called the fulcrum: as such we shall first consider them.

1. LEVERS WITHOUT WEIGHT IN THE BEAM.

96. *The First Kind.*

Fig. 42.



$$P : W :: BC : AC$$

or $3 : 9 :: 2 : 6$

$$\therefore (5)\ P \times AC = W \times BC$$

$$\text{from which we have } P = \frac{W \times BC}{AC} = \frac{9 \times 2}{6} = 3$$

$$W = \frac{P \times AC}{BC} = \frac{3 \times 6}{2} = 9$$

$$AC = \frac{W \times BC}{P} = \frac{9 \times 2}{3} = 6$$

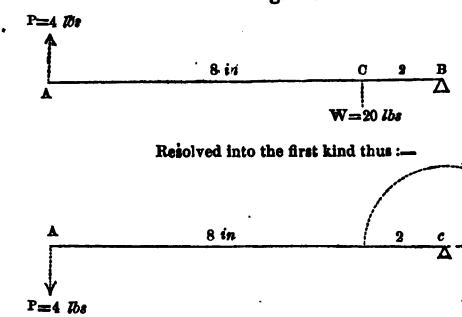
$$\text{and } BC = \frac{AC \times P}{W} = \frac{6 \times 3}{9} = 2$$

Of this kind of lever are the poker, when used in poking a fire, a handspike in raising a stone, &c.; also pincers, scissors, &c.

The Second Kind.

97. It is best to resolve the second and third kinds of levers into the first kind; for whether, in the following figure, a weight draws downwards with a force of 20 lbs. at

Fig. 43.



C, or with a force of 20 lbs. at a distance equal to B C beyond the fulcrum of the lever produced, as at *b* (in the *resolved*), will not signify, so that we reverse the direction of the power also. In like manner, if we reverse a sign in an equation, we must reverse all. By doing so in this case, we resolve the second kind into the first as under.

Using, therefore, the formula for the first (96), we say:--

$$P : W :: bc : ac$$
$$\text{or } 4 : 20 :: 2 : 10$$
$$\therefore P \times ac = W \times bc$$
$$\text{and } P = \frac{W \times bc}{ac} = \frac{20 \times 2}{10} = 4$$

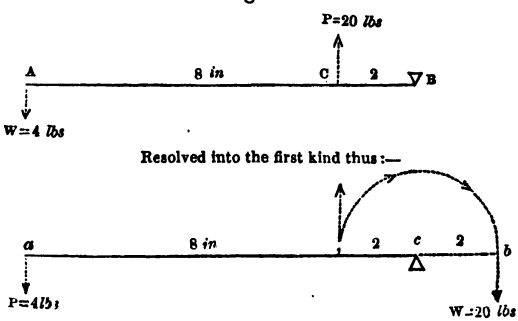
&c. &c.

Of this kind is the oar, the blade being the momentary fulcrum, the crutch representing the place of the weight, and the hand the power.

The Third Kind.

98. In this, when resolving, we have only to interchange the letters P and W, *calling the power weight, and the weight power,* for the occasion. That is to say, in the first figure that which stands at A as "W=4 lbs." will be altered in the second figure, at *a*, to "P=4 lbs.;" and that which, in the first, stands at C as "P=20 lbs.," will, when resolved in the second figure at *b*, be "W=20lbs."

Fig. 44.



So that the positions of the actual forces themselves will not be altered, *but only the letters change places in order to bring all levers under the one formula given at* (96) *for the first kind;* for what can it signify whether a weight

of, say 20 lbs., draws upwards at c, or downwards at an equal distance beyond the fulcrum at b, so that we reverse the fulcrum?

The definitions at (94) refer to the ordinary designation of the three kinds, and not to them when *resolved*.

Using again the formula for the first kind (in order to simplify), and retaining the positions of P and W as in the first and second kinds, although in reality reversed in signification, we have as before (96):—

$$P : W :: bc : ac$$
$$\text{or} \quad 4 : 20 :: 2 : 10$$
$$\therefore P = \frac{W \times bc}{ac} = \frac{20 \times 2}{10} = 4$$

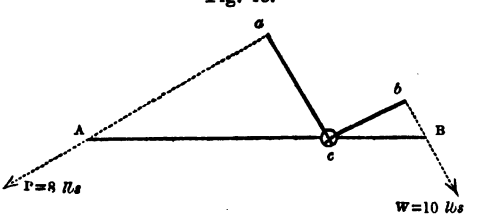
&c. &c.

Of this kind are sugar-tongs, fire-tongs, &c.; also the action of the muscles in animals; the human elbow; the holding of a fishing-rod; the safety-valve, &c.

99. Some authors speak of bent levers as a fourth kind, but they differ in principle in no respect from the straight lever, into which they are easily resolved; see Fig. 45, the line a A being the direction of the force.

100. We have hitherto considered levers as acted upon at right angles to the bar, but when the forces are applied to the end of the lever, obliquely, we proceed thus:—

Fig. 45.



Suppose, in the above figure, the power be applied in

the direction A P, and the weight in the direction B W, produce the line P a, and let fall a perpendicular, as $c\,a$, from the fulcrum upon it: do the same with W b, and we get thereby the points a and b.

In the figure we may suppose $a\,c\,b$ to be a bent lever.

Then, in the bent lever, the two forces are inversely as the arms $c\,a$ and $c\,b$; or, as again in the first kind (96): —

$$\begin{aligned} &P : W :: bc : ac \\ \text{or}\quad &8 : 10 :: 2{\cdot}4 : 3 \\ &\therefore P \times ac = W \times bc \end{aligned}$$

$$\text{and } P = \frac{W \times bc}{ac} = \frac{10 \times 2{\cdot}4}{3} = 8 \text{ lbs.}$$

By this means we bring all levers which are without weight under a simple and single equation.

It is worthy of notice that —

In the first kind, there is a gain of power when the power is less than the resistance of weight, but a loss of power when the power is *necessarily* greater than the weight.

In the second kind, there is always a gain of power, because the power must *always* be less than the weight.

In the third kind, there is *always* a loss of power, since the power must always be greater than the weight.

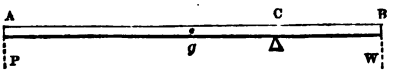
2. THE LEVER HAVING WEIGHT IN THE BAR, IT BEING UNIFORM THROUGHOUT.

101. *The First Kind.*

In Fig. 46, let A C = 8 inches
B C = 4 „
C g = 2 „
P = 10 lbs. (the power)
w = 6 „ (weight of bar)

Required W, the weight or resistance at B.

Fig. 46.



Now, in this case we have to consider two separate equations of "moments" (95).

First, there is that of a lever of the first kind, as if it had no weight;

$$\text{or (by 96) } P : W :: BC : AC.$$

But it is evident that some addition must be made to this formula, from the following reasons:—

The whole length of the bar weighs 6 lbs.; its centre of gravity will therefore be at its centre g. But a part of the bar is supported by the fulcrum C, and the amount thus supported will evidently be equal to twice B C, or $\frac{2}{3}$ of the bar, leaving $\frac{1}{3}$ of the bar, at some distance from the fulcrum towards A, requiring some counterpoise at B.

First, then, we have

$$P : W :: BC : AC$$

or, by the question, $10 : W :: 4 : 8$

$$(5) \text{ or } W = \frac{10 \times 8}{4} = 20$$

Secondly, we have to compensate the $\frac{1}{3}$ the lever (lying next to A) by a weight at B. Now this $\frac{1}{3}$ or 2 lbs. (the lever weighing 6 lbs.) would have its centre of gravity at $\frac{1}{2}$ that length, or at $\frac{1}{6}$ the length of the whole bar from A; or we should have 2 lbs. as a power at a distance of 6 inches from the fulcrum C, and, B C being already given as 4, we have the second equation thus:—

$$2 \times 6 = BC \times \text{the compensating weight (which call } x);$$

$$\therefore x = \frac{2 \times 6}{BC\,(=4)} = \frac{12}{4} = 3 \text{ lbs.}$$

Hence 20 lbs. (the weight of W) added to 3 lbs. (the weight of x) = 23 lbs., which is the gross weight required to counterpoise the lever itself together with the 10 lbs. at A.

Now, adding these two equations together, we get

$$W + x = \frac{10 \times 8}{4} + \frac{2 \times 6}{4} = \frac{92}{4} = 23 \text{ lbs.}$$

But allowing W to represent the *gross* weight hanging at B, we have therefore the usual equation for the lever of the first kind without weight, with an addition in consideration of the weight of the bar itself, as follows: —

Instead of P : W :: B C : A C (as at 96)
(or by 5) $P \times AC = W \times BC$
we have $(P \times AC) + (w \times Cg) = W \times BC$

$$\text{or } W = \frac{(P \times AC) + (w \times Cg)}{BC}$$

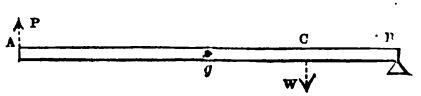
or, taking the quantities from the above question,

$$W = \frac{(10 \times 8) + (6 \times 2)}{4} = \frac{92}{4} = 23 \text{ lbs. as above.}$$

Whence any other term in the equation may be found (5); and the same applies to the second and third kinds, which need no further explanation.

102. *The Second Kind.*

Fig. 47.



$$P \times AB = (W \times BC) + (w \times gB)$$

∴ To find the power (5):

$$P = \frac{(W \times BC) + (w \times gB)}{AB}$$

To find B C:

$$B\,C = \frac{(P \times A\,B) - (w \times g\,B)}{W}$$

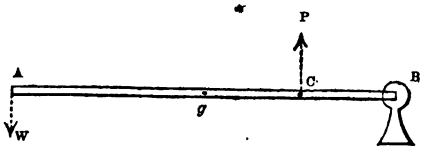
To find the weight:

$$W = \frac{(P \times A\,B) - (w \times g\,B)}{B\,C}$$

&c. &c.

103. *The Third Kind.*

Fig. 48.



$$(P \times A\,B) + (w \times g\,B) = W \times B\,C$$

∴ To find the power (5):

$$P = \frac{(W \times B\,C) - (w \times g\,B)}{A\,B}$$

To find B C:

$$B\,C = \frac{(P \times A\,B) + (w \times g\,B)}{W}$$

To find the weight:

$$W = \frac{(P \times A\,B) + (w \times g\,B)}{B\,C}$$

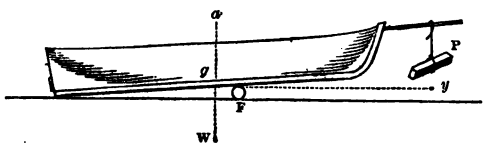
&c. &c.

104. A very useful method of ascertaining at sea the approximate weight of a spar or boat, or mass of metal,

&c., having its length greater than its breadth, is as follows: —

Ex. — Being desirous of knowing the weight of a boat, proceed as the accompanying figure indicates: —

Fig. 49.



Place a roller under the keel until the boat is fairly balanced upon it. Make a mark g (exactly over the centre of the roller) upon the boat's keel. Call this the centre of gravity; the true centre of gravity being somewhere in the vertical line $a\,g$ (94), for all the weight of the boat may be said to concentre there. Move the roller (F being its centre) a few inches forward, and sling a pig of iron to the boat's bowsprit, shifting it until it exactly counterpoises the after part of the boat. *When the boat is upon an even keel,* measure $\mathrm{F}\,y = 12$ ft. 6 in., and $\mathrm{F}\,g = 9$ in. The boat is thus *weighed;* for, according to the general law for levers, $\mathrm{P} : \mathrm{W} :: \mathrm{F}\,g : \mathrm{F}\,y$ (96). The iron pig weighed 100 lbs.

$$\therefore \mathrm{W} = \frac{\mathrm{P} \times \mathrm{F}y}{\mathrm{F}g} = \frac{100 \times 12{\cdot}5}{{\cdot}75} = \frac{1250}{{\cdot}75} = 1666\tfrac{2}{3} \text{ lbs.} = 15 \text{ cwt. nearly.}$$

An irregularly shaped long piece of iron, such as a spindle or shaft, or even an anchor, may be weighed with great approach to accuracy; for it is obvious, as regards the latter, that although the two palms might not counterpoise *laterally,* any weight might be placed on one palm so as to effect this, it being afterwards deducted from the weight found by the above process.

On examining the figure it will be seen that in reality $g\,y$ corresponds with a lever, of which F is the fulcrum, and whether the weight rest above g, or is suspended below it, is immaterial.

A mass of any shape may thus be weighed by lashing it to a plank, and the weight of the plank can be afterwards taken and deducted.

PARALLELOGRAM OF FORCES.

Anything which produces, or which has a tendency to produce, motion in a body, may be called a force.

Thus gravity is a force which tends to move all bodies towards the earth's centre (94). Forces are natural or artificial. Those are natural which result from direct physical laws, such as gravity, centrifugal force, &c.; and those which arise from impacts, or obstructed motions of bodies, &c., are considered artificial. Our present purpose is with the latter.

105. If two bodies of equal weight approach each other in a direct line, with equal velocities, their meeting will produce a state of rest if no other forces are in operation upon them. This is called an equilibrium of forces.

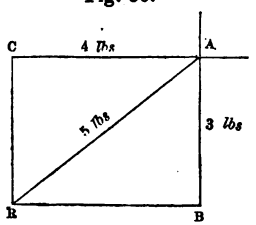
But if bodies of different weights or velocities approach in a straight line, the result will be in accordance with such difference, and a new motion will arise in the same straight line.

106. But if they do not approach in the same straight line, a new motion will be generated in a new direction.

107. If, for example, a body weighing 3 lbs. meet at right angles another body weighing 4 lbs., the new compound force arising from the two forces will result in a certain direction called the *resultant*, and the amount of new force so generated will be called the *magnitude* of such resultant. If in the figure a force of 3 lbs. move in

the direction of A B, and the other in the direction of A C, by drawing A B = 3 equal parts from any side, and A C

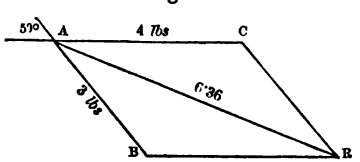
Fig. 50.



= 4 such parts, and making C R parallel to A B, and B R parallel to A C, we shall form what is called the parallelogram of forces, in which A R would be the resultant, and on the same scale would measure 5 lbs.

108. But if the two forces meet at any oblique angle, say at 50°, they will then give rise to the following form of parallelogram:—

Fig. 51.



in which A R is the resultant, and its length the magnitude, in this case amounting to 6·36 lbs. It thus becomes a mere question of trigonometry.

109. In calculating the triangle to find A R, we have two sides given, and the included angle B or C; for C A B = 50°, and C R B = 50°, and, because the opposite angles of a parallelogram are equal (Euc. I. 34), 360° − (50+50 =) 100° = 260° = the sum of the two angles, C and B, and as

they are equal in the parallelogram, C = 130°. Then by rule (60): —

As A C + A B = 7 . .	ar co	9·154902
Is to A C − A B = 1 . . .		0·000000
So is tan ½ sum $\frac{180-130}{2}$ = 25°		9·668673
To tan ½ diff 3° 48′ 30″		8·823575
∴ greater ∠ B A R = 28 48 30		

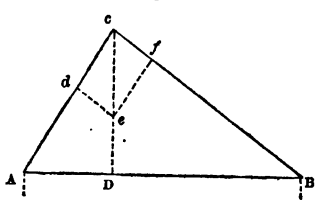
By rule of sines (45): —

As sin B A R 28° 48′ 30″ . .	ar co	0·317060
Is to side B R 4		0·602060
So is sin ∠ B 130°		9·884254
To side A R = 6·36		0·803374

110. Where more than two forces are acting upon one point, they are resolved by operating with two of them, and taking the resultant as a new force in combination with the third, and so on.

As an instance of the application of the law of forces, let a roof, A B C, weighing 10 tons, stand upon two walls, A and B. Required to know from the figure the pressure on the top of each wall.

Fig. 52.



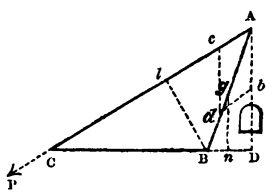
Draw $c\,e = 10$, and complete the parallelogram, and the sides $c\,d$ and $c\,f$ will each represent fractions of 10=

to the respective weights on A and B, and the segments A D, D B will be in proportion respectively to the lateral thrust upon each wall or upon the ends of the tie beam A B; so that, of the whole weight of the roof, the weight resting on each wall is inversely as the lengths of the sides of the roof.

The following example may be given as another illustration of the great importance of a knowledge of trigonometry, the lever, &c.

Required, from the following figure, the tension of a chain which supports a pair of shears capable of lifting a weight of twenty tons.

Fig. 53.



1st. *To find the tension of* A C *by the law of forces.*— Given $\angle$ *c* A *b* and $\angle$ *d* A *b*, and the weight twenty tons. Draw the parallelogram A *b d c* A, making A *b* = 20 from any scale of equal parts, make *b d* parallel to A *c*, and *d c* parallel to A *b*. Then A *d* divides the parallelogram into two triangles, in which, in $\triangle$ A *c d*, the side *c d* = A *b* = 20, and we wish to find A *c*, which will represent the tension of the chain A C.

$$\text{Let } \angle c\,\text{A}\,d = 38^\circ\ 3'$$
$$\angle d\,\text{A}\,b = 22^\circ\ 24'$$

Then, because the right line A *d* falls on the two parallel

right lines A c, d b, the alternate angles are equal (Euclid I. 29); therefore $\angle$ A d c = 22° 24′ (24).

Then, by rule of sines (45),

As sin $\angle$ c A d 38° 3′	.	ar co 0·210173	
Is to side c d 20°	. . .	1·301030	
So is sin $\angle$ c d A 22° 24′	. . .	9·581005	
To side A c . . .		= 1·092208	= 12·37 Tons.

N.B.—If the angles had not otherwise been given, A B, A C, B C being known would have given them (62).

2nd. *To find the tension of* A C *by the rules for the lever.* — In this case it will be obvious, that whether 20 tons be suspended from A towards D, or from D itself, its effect would be the same, and because two forces are acting, one on each side of A B, so, to preserve equilibrium, the axis of moment must be the point B. This represents, therefore, the fulcrum of a lever of which B D is one arm, sustaining at D its W = 20 tons, and the other arm is obtained by resolving the oblique direction of the force P into a rectangular one (100). Let, therefore, the perpendicular upon the side A C be 9·86 feet, and B D = 6·1 feet. Now, because the moment of power must equal the moment of weight (95), we have for levers of the first kind (96),

P : W :: B C : B A; or, from different lettering in the above figure,
P : W :: B D : B l
or P × B l = W × B D
or $$P = \frac{W \times BD}{B\,l} = \frac{20 \times 6{\cdot}1}{9{\cdot}86} = 12{\cdot}37 \text{ tons.}$$

The above is supposing the weight of the shears had not been taken into account, but if the shears weighed 1½ ton, the formula would have been

Tension of A C or P × B l = W × B D + (1·5 × B n)

or $$P = \frac{20 \times 6{\cdot}1 + (1{\cdot}5 \times 3{\cdot}06)}{9{\cdot}86} = 12{\cdot}84 \text{ tons.}$$

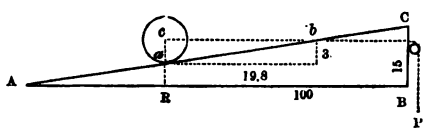
The distance B n is obtained in the following manner: —

Supposing A B to be uniform in weight, its centre of gravity would be at g its centre, and a perpendicular from g on B D would fall at n; and because g is the centre of A B, and $g\,n$ is parallel to A D, the triangles g B n and A B D would be similar; and because B g is the half of B A, therefore B n would be the half of B D.

INCLINED PLANE.

111. Suppose the ball in the figure weighs 20 cwt., and the horizontal length of the triangle is 100 yards, and the height 15 yards: required the vertical and horizontal pressure of the mass.

Fig. 54.



Draw the triangle A B C, whereof A C is the inclined plane, and B C its height, and from any point a (the place of the ball) lay off $a\,b$ = the weight 20; draw the parallelogram, and $c\,b$ will represent in magnitude 19·8 cwt., the vertical pressure, and $c\,a$ the horizontal pressure, 3 cwt. So that 3 cwt. suspended at P, passing over a pulley and fastened to the mass, would prevent its rolling. Or, by calculation, we have this general formula: —

$$\begin{aligned} &\text{P} : \text{W} :: \text{height of plane} : \text{base} \\ \text{or,}\quad &x : 20 :: \quad 15 \quad : 100 \end{aligned}$$

$$(6)\qquad \therefore x = \frac{20 \times 15}{100} = 3 \text{ cwt.}$$

N.B. Putting x to indicate the term sought, had, for instance, the height of the plane been unknown, we should have called it x, and have used the formula thus: —

$$\begin{array}{l} \text{P : W :: height of plane : base} \\ 3 : 20 :: \quad x \quad : 100 \end{array}$$

$$(6) \qquad \therefore x = \frac{3 \times 100}{20} = 15 \text{ yards.}$$

THE PULLEY.

This subject will, for obvious reasons, be treated quite practically, and irrespective of either the rigidity of the rope or friction in the block.

112. It is usual to divide the consideration of pulleys into three systems, all immediately derived from, and resolvable into, the *simple purchase*, as under.

N.B.—The fixed sheave a (Fig. 55) is merely used to give direction to the power P, and forms no part of the purchase.

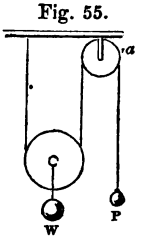
Here we have P : W :: 1 : 2

$$(5) \quad . \quad . \quad . \quad . \quad . \text{ or, } P = \frac{W}{2}$$

Ex.—What power at P will sustain 100 lbs. at W?

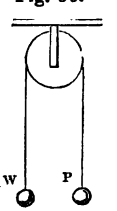
$$\text{By the above equation, } P = \frac{100}{2} = 50 \text{ lbs.}$$

Fig. 55.



Single Purchase.

Fig. 56.



Whip.

The Whip.

This is no "purchase," as the weight equals the power.

The First System.

N.B.—The block *a* is only to give direction to the power P, and forms no part of the purchase (Fig. 57).

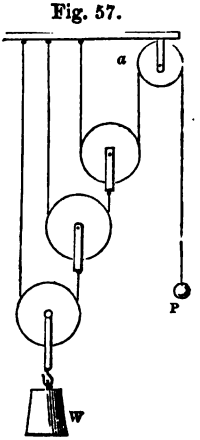
Let n = the number of pulleys,
then P : W :: 1 : 2^n

$$\text{or, } P = \frac{W}{2^n}$$

Ex.—What power at P would sustain 100 lbs. at W, there being three pulleys?

$$P = \frac{100}{2^3} = \frac{100}{8} = 12\tfrac{1}{2} \text{ lbs.}$$

Fig. 57.



The Second System.

Let n = number of ropes in lower block,

$$P : W :: 1 : n$$
$$\therefore P\,n = W$$

Ex. 1.—In a gun-tackle purchase, what power at P will sustain 100 lbs. at W?

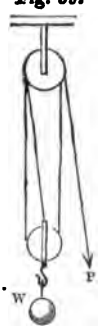
$$\text{If } P\,n = W$$

$$\therefore P = \frac{W}{n} = \frac{100}{2} = 50 \text{ lbs.}$$

Fig. 58.

Gun-tackle Purchase.

Fig. 59.



Luff-tackle Purchase.

Ex. 2.—A luff-tackle is required to lift 21 cwt.: what power at P will counterpoise it?

$$\text{If } P\,n = W$$

$$\therefore P = \frac{W}{n} = \frac{21}{3} = 7 \text{ cwt.}$$

The Third System.

Let n = the number of pulleys,

then $P : W :: 1 : 2^n - 1$

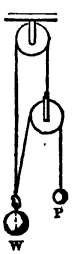
or, $P = \frac{W}{2^n - 1}$

Ex.—A burton is hoisting a weight of 100 lbs.: what power at P is required to sustain it?

$$\text{If } P = \frac{W}{2^n - 1}$$

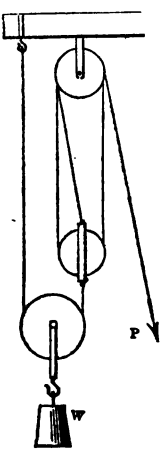
$$\therefore P = \frac{100}{4-1} = \frac{100}{3} = 33\tfrac{1}{3} \text{ lbs.}$$

Fig. 60.



Single Burton, or Whip-Burton.

Fig. 61.



The Runner.

Where a ship is short-handed, or a heavy strain is required, such as in purchasing the anchor when the capstan is damaged, or the staying of a mast, a runner is used with advantage over the ordinary luff or treble-block purchase.

Ex.—Suppose a purchase of ten tons be required to act at W, what strength would be necessary at P to sustain it?

1st. The luff-tackle (Fig. 59) would support half, and the standing part of the runner half also.

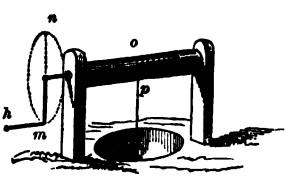
Then, by Ex. 2, p. 116, $Pn = W$,

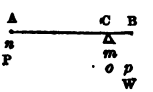
or, $P = \frac{W}{n}$ = by the question $\frac{5 \text{ tons}}{3} = 1$ ton $13\frac{1}{3}$ cwt.

THE WHEEL AND AXLE.

113. As applied to practice, the lever acts only through short spaces, and works intermittently, while by means of the wheel and axle we apply the very same *principles* to produce a *continuous* power. Taking the common winch for drawing water, or for lifting materials by builders, &c., as an example,

Fig. 62.





let $m\,n = 16$ inches (being the diameter of the circle described by the handle h in working), and let $o\,p = 5$ (being the diameter of the axle).

If on a straight line we lay off $m\,n =$ A C (as above), and $o\,p =$ C B, then A B represents a lever of the first kind, of which C is the fulcrum point. Let W = 160 lbs.

Then (96) P : W :: B C : A C

or P : 160 :: 5 : 16

$$\therefore P = \frac{160 \times 5}{16} = 50 \text{ lbs.}$$

N.B. All calculations on the wheel and axle are founded on this rule.

THE SCREW.

114. The thread of the screw being considered as the hypotenuse of a triangle, and the distance between the threads as the perpendicular, the circumference of the base would be the circumference of the cylinder out of which the screw is cut. Such a triangle may be supposed to be wrapped round the cylinder, and if the height or perpendicular represents more than one such distance between the threads, cutting a triangle out of paper and wrapping it round a cylinder (such as a black-lead pencil) will plainly illustrate that the calculations for the screw are essentially those of the triangle. (See "Screw Propeller," 167).

PART II.

SECTION I.

STUDY OF STEAM.

THE MARINE ENGINE FULLY DESCRIBED IN CONNECTION WITH PRINCIPLES.

115. In commencing a study on a new subject, it is well to endeavour to discover in it a similarity to some other subject already understood. In studying a new language, for instance, if we can trace any connection between it and our own "vernacular" in its declensions, conjugations, &c., we begin under advantages. This applies equally to the study of steam, in which a large and apparently complicated machine is presented to us, offering for our consideration the attributes of strength, precision, and apparently almost intelligence. We are, therefore, almost naturally inclined to ask if it can bear any analogy to animal bodies? Let us see. An animal, apart from the mysterious "vital principle," so called, is a piece of mechanism, composed of cords and levers and springs, its action depending on a constant supply of an invisible "air," which, on being received into one part of the body (the lungs), undergoes chemical decomposition, and by means of various processes, involving the use of *valvular* appliances, action is produced, and sustained with regularity so long as this "breath of life" is supplied; but stop this, and the life stops also. It is precisely so with the steam engine

(so far as regards general principles). For, in like manner, an invisible elastic fluid enters the cylinder (the "*lungs*") of the engine; it is there (mechanically) decomposed, and various subordinate appliances, all tending towards the heightening of the effect produced by such decomposition, perform their offices with precision and regularity, until, as in the animal body, *action* is produced, and sustained so long as the aërial fluid is supplied; but stop that supply, and the action ceases altogether.

Hence we see that in either case there are parts of these "machines" which may be considered *vital* or essential to the performance of certain indispensable functions, and which are analogous; and, dropping metaphor, we perceive that a proper understanding of the cylinder, or first receptacle of the steam, should be our first consideration.

In order to make the use of this plain to the uninitiated, we will, for the present, dispense with a disquisition on the properties of steam, and merely consider it as what it really is in effect, viz. a peculiar *mechanical combination of water and heat.* Nor will we as yet even venture upon the question of the properties of heat beyond the requisites of a proper notion of condensation.

In the steam engine we see an effect produced, accompanied by a movement of its various parts, and we should, under ordinary reasoning, have to trace the cause of such motion through the several parts up to its source, were it not that all the motions are simultaneous, and due to the one operation, viz. the admission of steam.

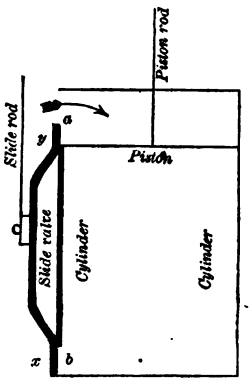
We therefore begin our examination by roundly assuming steam to be the apparent "cause" of motion, or the prime mover of the machinery; or, in common terms, we let steam into the engine in order to "*set it going.*" It is not yet necessary for us to know that scarcely two engines are made alike; that every maker has his peculiar form of machinery; that some engines have fixed cylinders,

while others are made to oscillate; that in some a cylinder is placed horizontally, and in others vertically; that there are side levers, steeple engines, single-acting, double-acting engines for paddle-wheels; engines for screws, mines, wells, &c., &c. *For all are upon one and the same settled principle,* and the parts to be found in the simplest engine are to be recognised in the most complicated, — just as in the line-of-battle ship the sails are upon the same principle as those which govern the fishing-boat; but the former has a more elaborately contrived mast, has its sails in greater divisibility, and is fitted with a variety of spars and ropes for certain purposes not required in the latter.

The first attempt at illustration will show that the passage of the steam into the cylinder is very much like the act of animal respiration; for if the steam passage be too contracted in the machine, difficulty in its operation of *breathing* is detected in the *wheezing* noise produced in the cylinder. And again, a superabundance of steam causes unnecessary "plethora" and waste of power, and increased unnecessary wear and tear of the parts, just as our breathing too much oxygen would shorten our lives by prematurely *wearing us out.* Therefore, the first care of the engineer is to contrive a proper-sized "throat" for the whole apparatus, and from the regulation of the capacity of this throat for the transmission of steam by means of the "throttle valve," there results a healthy or inadequate state of things. Various contrivances for the regulation of the area of this "steam pipe" are used with beautiful efficiency, and when we have to consider the question of expansion (140), it will be properly explained.

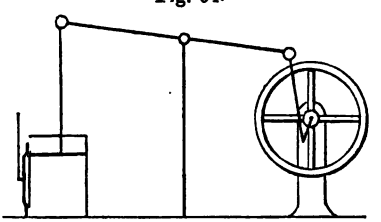
116. In describing the use of the cylinder, it will be more convenient to consider the steam in its normal state as a mere elastic force. The following skeleton diagram will convey a notion of the cylinder in its simplest form:—

Fig. 63.



The piston (shown here by mere outlines) works freely, but steam-tight, up and down the cylinder, from the following cause. Suppose, in the figure, *a* to be an aperture open to the steam-pipe; the steam, having a pressure greater than that existing below the piston (119), rushes into the cavity above the cylinder, forcing the piston downwards. A mechanical contrivance (not yet shown) acts upon the slide valve by means of the "slide rod" *during the descent of the piston*, and raises the slide valve *y x*, so that by the time the piston reaches the bottom of the cylinder the aperture *a* becomes closed, and the lower aperture *b* is opened, immediately on which the steam from the steam-pipe rushes into the lower portion of the cylinder, and forces the piston upwards until the motion of the slide again opens *a*; and thus an alternate up and down motion of the piston ensues, which, by simple mechanism, is converted into a revolving motion (Fig. 64),

Fig. 64.



turning the shaft of the engine, and therefore the paddle-wheel or screw attached to it. There is in both the upper and lower portions of the cylinder a pipe provided for the

exit of steam which has done its work, and by its escape from the cylinder no impediment to the alternate motion of the piston remains.

When steam is used in this manner, it is (not very accurately) said that the engine works at high pressure; and it is this, which, from requiring no condenser, is most applicable to the railway locomotive.

117. The waste steam is not allowed to escape at once into the air, but is conducted to the chimney, where it exerts the power of increasing the upward current by passing up a "blast-pipe."

Another development of action in the cylinder may be described as caused by

CONDENSATION.

118. We have spoken of steam as a peculiar mechanical combination of water and heat. Now, if we hold a piece of cold metal in contact with steam, the metal, being a good conductor of heat, has the power of absorbing among its atoms (133) the heat which the water required to form it into steam on boiling; and the water, not being absorbable by the metal, remains on its surface. But there is this important consideration, viz.: water combines with heat, until, in the form of steam, it occupies a space of 1700 times its former bulk, so that, consequently, the heat being again withdrawn, the steam diminishes in bulk to its 1700th part. In round numbers, we may say a cubic foot of steam suddenly thus becomes a cubic inch of water. It is this process of reconverting steam into water which is called "condensation," and it is the reverse operation from that of heating water to the boiling-point, and thus causing (in the formation of steam) an accumulation of expansive force, which gives motion in the steam engine.

We take advantage of this condensation in its forming what is called a *partial vacuum* in the parts of the cylinder alternately above and below the piston, and it is also this alternate *sudden displacement* of matter which causes the alternate pressure on either end of the piston, which, without the active pressure of any steam at all, would occur from the nature of the vacuum itself. But we have spoken of cold metal as condensing steam, merely for simplicity of illustration. In the steam engine cold water is a much more manageable, though less powerful agent: the heat of the steam in the cylinder is suddenly absorbed by fine jets of water thrown into a receiver of the expended steam called a condenser, and it is here that the sudden diminution of bulk in the steam is effected, causing the alternate pressure on the piston referred to.

The nature of a vacuum ought to be clearly understood. The following description will greatly assist the young officer, and at the same time acquaint him with the uses and principles of certain bent tubes used with mercury, which are to be seen attached to steam engines.

ATMOSPHERIC PRESSURE.

119. A vacuum is a perfectly void space. Among engineers the term is more properly "partial" vacuum, for a "perfect" vacuum can scarcely exist on the surface of our earth.

If it were possible to erect a tube (having a sectional area of one square inch) upon any part of the earth, supposing this tube to be long enough to reach up through the thickness of the atmosphere (which is said to be about forty-five or fifty miles), the air contained in such tube would, in round numbers, weigh about 14$\frac{3}{4}$lbs. If, further, we take a tube of the same diameter and put water

in it to the weight of 14¾lbs., the level of the water in the tube (if standing upright) would be at about thirty-four feet above the bottom of the tube. If, again, we take a similar tube, 14¾lbs. of mercury would fill it up to about 30 inches. The accuracy of these numbers would be affected by temperature, &c.; therefore, for practical purposes, we assume atmospheric pressure to be 15lbs. upon every square inch, so that *every pound of pressure equals two inches height of mercury.* But we must better understand what pressure is, and how it is exerted.

If we take the bell-glass of an air-pump, and, it being open at the top, close the aperture with the hand while the air is being exhausted from inside the glass, on attempting to remove the hand, it will be found impossible, because, supposing the diameter of the open part of the bell-glass under the hand to be only three inches, it would have an area of seven square inches, upon each of which would rest a weight of say 15 lbs., so that upwards of 100 lbs. of air would be resting its weight upon the hand; but as soon as we readmit the air into the glass, the air re-exerts a counter-pressure *under* the hand, and equilibrium of pressures being nearly restored, the hand can be taken away. Now, if we further suppose the hand to have been acted upon precisely as the piston is in the steam engine, we have here an illustration of what is meant by *equilibrium of pressures*, which is a definition of the principle causing motion in the steam engine; for the balance of pressure, alternately in favour of either side of the piston, causes, as already observed, the up and down motion of the piston rod, which moves the connecting rod, crank, and axle or spindle. Thus, when we speak of *vacuum*, we only mean the side of the piston at which there is the *less amount* of pressure. The ordinary barometer measures such balance of pressure. In the weather barometer the vacuum is a constant quantity, because the

space above the higher surface of the mercury is protected from any atmospheric pressure by the glass being sealed, and the atmosphere therefore presses upon the open surface only, and forces the mercury up almost to the top of the empty tube, until the weight of the mercury thus raised in the tube counterpoises the weight of the external air pressing on the open surface of the mercury in contact with the air; and this average height is, as before observed, about thirty inches.

120. In the steam engine, barometer tubes, or *gauges*, are fixed, communicating with the cylinder, condenser, boiler, &c., to measure pressure, for the pressure caused by steam in the boiler may be precisely compared with that caused by the weight of the atmosphere; but we do not use mercury as a necessity, for, attached to the cylinder, we have a clever little apparatus, called an indicator, in which, instead of a counterpoise of mercury, we employ a spring to ascertain the power of steam used. (See "Indicator," 148.)

121. It may be remarked here, that the heights of fluids under pressure vary in proportion to their densities ("Specific Gravity," 173). For instance: if a cubic foot of water weighs 62½ lbs., and a cubic foot of mercury, under the same pressure, 848 lbs., the one is about 13½ (13·624) times as heavy as the other; and therefore 34 feet, the height of a column of water to counterpoise atmospheric pressure, divided by 13½, gives about 30 inches, the rough average height of mercury when similarly circumstanced.

In describing the cylinder, we have in reality explained the *whole* principle upon which motion is conveyed to the spindle or shaft of the machine, and by which the paddle-wheel or screw is made to revolve; but there are appliances to the steam engine which render the working of the cylinder more uniform and accurate, such as the air-pump

for improving the vacuum, pumping out condensed water, air, &c.; the condenser for still further facilitating the same operation; the hot well for receiving the feed-water prior to its entering the boiler to renew the supply of water, as it is converted into steam, and used in the cylinder of the engine; the feed-pipe and pump for securing a supply of water for the boiler; the governor for cutting off the steam from the machinery as soon as too great speed is attained; expansion gear for regulating the supply of steam to the cylinder, so as to economise steam, and thereby fuel. There are various beautiful contrivances for the movement of valves throughout the engine; these will be explained under their proper heads.

We began our description by supposing that steam had been let into the engine through the "throttle valve." We will now explain the manner in which the steam is generated and produced in the

BOILER.

122. We may call any receptacle for water, for use over a fire, a boiler; but that which forms so important a part of the Marine Steam Engine has peculiarities the principles of which demand clear explanation.

As water is a very bad conductor of heat, the operation of boiling a large quantity of water would be a tedious process, unless care were taken to bring a large surface of the water in contact with the heat at the same moment. It is moreover important that steam ships should be able to get up their steam in as small space of time as possible, for many emergencies may arise by which the ship may be endangered unless furnished with the means of progression.

In sailing vessels, the setting and trimming of the sails

does this in a very few minutes. In steam ships, however, much depends upon the skill displayed in the construction of the boiler, the grate, and the flues, as adapted to each other.

In the boiler we have, besides the peculiar form of it, distinct subjects for consideration, such as Fuel, Ventilation, Combustion, and Evaporation.

FUEL.

123. We are too apt to consider that coal, or its substitute, is the only fuel necessary to insure the production of steam: this is partly because we see it taken on board ship, and stowed in bunkers as such. But in reality coals form only a small part of the fuel consumed in the furnace of a steamer, for between 12 and 20 lbs. weight of atmospheric air are required in the combustion of every pound of coal, or about as much air as would be contained in an ordinary room 12 feet square; and this may be doubled when we think of the quantity of air still further necessary to dilute the carbonic acid gas formed in combustion, which gas, if undiluted, would instantly put the fires out.

The production of heat requires one element which it is often difficult to procure in sufficient quantity on board ship, viz. oxygen (which forms 21 per cent. of atmospheric air); and the arrangement of the furnaces, ash-pit, and flues is always a primary object with the engineer, as not only is the boiling of the water influenced by the facilities combined in these portions of the apparatus for a proper supply of air, but considerable waste of fuel and cause of expense may result from negligence or ignorance in the arrangement. As engines on shipboard are always placed as low in the hold as possible, it follows, from there being only access to the atmosphere in the "ceiling," as we may

call the deck over the engine-room, that artificial means of furnishing a proper supply of atmospheric air, not only for the sake of its oxygen, but for the purpose of ventilation, are necessary.

VENTILATION

124. Is important, not only on account of the demands for air for the grate, but in order to replace the heated air with rapidity enough to insure a temperature sufficiently low for the engineers themselves to work in without inconvenience. This supply of air is partly afforded by means of canvas tubes, with open ends, which are let down in the engine-room, and are called "windsails." The lightness of the heated air causes it to ascend from all parts of the engine-room, and favours the downward rush of colder air from above to replace it; but, from what has been said, the main cause of the need of so large a supply appears to be the decomposition of the air which enters the furnaces by the ash-pits,—in fact, the consuming of the air as well as the coal. The proper ventilation of marine engine-rooms yet remains a question of difficulty. But let us now consider what *is*

COMBUSTION.

125. It is, according to popular notions, the *burning* of something, but burning means *destroying:* with the chemist, however, it is only the separation of the various elementary ingredients of various bodies by means of heat: as an instance, we are said to "burn" paper when we "set light to it;" but what do we then do? The paper is formed of a woody substance called "lignin," and contains about one-half its weight of carbon, and the remainder is

either water or the elements of water (*i. e.* oxygen and hydrogen), so that 100 parts paper would furnish about 50 parts carbon, and about $4\frac{1}{2}$ parts hydrogen, and $45\frac{1}{2}$ parts oxygen. Now, when we subject paper to heat, we raise its temperature until reaching 700° or 800° Fahrenheit; the paper becomes decomposed, that is, its elements or ingredients separate: the carbon unites with the oxygen set free both from the paper and as obtained from the atmosphere, and forms carbonic acid gas (that which we drink in effervescing beverages), and the hydrogen, forming water by addition to a portion of the heated oxygen, is given off as such, and is easily detected by holding a cold metal or a cold glass over the flame, when a film of steam will be momentarily seen. But further, sometimes the paper does not *all* disappear, and little active sparks run about the thin black tissue left, until, rapidly and suddenly, one by one, they "go out," for want of sufficient heat to effect total combustion. In like manner, when we put coals on a fire, on applying heat a precisely similar decomposition takes place; only we may notice that, unless we get a high temperature, decomposition takes place *without perfect combustion.* In such case a large portion of the fuel, for want of combining power, goes up the chimney in the form of smoke and is wasted; while, if we had the means of applying *flame* to this carbon, mixed as it is with gases, it would burst into a bright sudden flame, and increase rapidly the activity of the fire. So that perfection of combustion in an engine furnace is known by the absence of smoke; for when carbon is perfectly consumed, and a proper temperature attained and maintained, carbonic acid gas is formed, and it invisibly escapes up the chimney, having performed its duty.

EVAPORATION.

126. It has already (p. 120) been stated that steam is a peculiar mixture of heat and water. The process which forms the combination is called evaporation. This evaporation takes place at all temperatures; as an example, if we place a drop of pure water on a thin slip of glass, and expose it to the air, it dries and disappears. This is caused by the formation of vapour, the water combining with the heat of the air, and is assisted by an electric action caused by the friction of the particles of the air passing over the surface of the water. If we place the same over a candle, the same thing occurs; only the compound of heat and water, which flies off in an invisible form in the first case, is called "vapour;" and in the latter, that which is dissipated by heat after ebullition or boiling, is called "steam." In either case the process is that of "evaporation." Now, if we drop water on the back of the hand, and expose it to a current of air, we have a sensation of increasing cold produced during the evaporation. This is because, as above stated, vapour and steam are a mixture of heat and water; and the taking away the heat from the hand to enable the drop of water to form vapour, is the sensible production of cold (cold is merely absence or diminution of heat).

127. Hence the production of steam involves several important considerations, and certain things have to be done in which such considerations must all tend to one object. It will easily be conceived, therefore, that the several parts of a boiler must be in proportion. In the first place the ash-pit must be so contrived as to allow a sufficient current of air to promote combustion; — there must be a due proportion between the entry of air into the furnace and the dimensions of the funnel as an escape for used-up gases or newly-formed ones. The heating surface

of the boilers must have a due proportion to the quantity of water to be converted into steam. The pipes for the supply of water to the boilers (feed-pipes) must be in proportion to those for the escape of steam; therefore, a certain equilibrium is thus to be maintained which gives uniform speed to the engine, by causing a uniform rate of evaporation. Then there arises the question of pressure in the boiler, as requiring sufficient strength in the plates to resist it; a proper arrangement of safety-valves to relieve this pressure when it approaches too near the estimated limit of prudence. There are contrivances, moreover, for ascertaining the height of the water in the boilers, lest the water become deficient, and a sudden supply of steam endanger the safety of the boiler; on the deficiency being too great, there is a danger of the bottom of the boiler becoming overheated, and of course softened, &c. All these are subject to rules which, in the following portions of this book, will appear under their proper heads.

Having, then, sufficiently explained the nature of the working of the machine as a whole, we shall consider the details of the principal parts.

SECTION II.

CALCULATION OF PARTS OF THE ENGINE.

HORSE POWER.

128. The power of an engine is commonly estimated by the number of pounds weight which could be raised one foot high in one minute by one horse. Watt defined this to be 33,000 lbs. Although this is not accurate, the number has been retained as an element in calculating the horse power of work.

129. *Nominal* or commercial horse power is the work expected from an engine as founded on the assumed pressure of steam, amounting to 7 lbs. per square inch of piston surface, and as used in tenders for contracts for Her Majesty's Navy in London; while in the Clyde it is $7\frac{1}{2}$ lbs. Therefore 14-horse power in London equals 15 on the Clyde.

The following is the Admiralty equation for *nominal* horse power :—

$$\text{Nominal horse power} = \frac{\text{area of piston in ins.} \times 7 \times \text{twice the stroke} \times \text{no. of revol. per minute}}{33000}$$

And supposing, for example, the diameter of the cylinder to be 60 inches, and the pressure 7 lbs., the length of stroke 3 feet, and the revolutions per minute to be 50 in number,

$$\text{the horse power would} = (60^2 \times \cdot 7854) \times 7 \times (2 \times 3) \times 50$$
$$\text{or} = 2827 \times 7 \times 6 \times 50$$
$$= \frac{5936700}{33000} = 180$$

Or thus,—

$$\text{Nominal horse power} = \frac{(\text{diameter cylinder in inches})^2 \times 2\ (\text{stroke}) \times \text{revolutions per minute}}{6000}$$
$$\text{or} = \frac{3600 \times 6 \times 50}{6000} = 180 \text{ (as before)}$$

To find the *actual* horse power :—

$$\text{Actual horse power} = \left(\begin{matrix}\text{area of piston} \times \text{actual pressure per in.} \\ \text{,, of piston surface}\end{matrix}\right) \times \left(\begin{matrix}\text{length of} \\ \text{stroke}\end{matrix}\right) \times \left(\begin{matrix}\text{no. of revol. per} \\ \text{minute}\end{matrix}\right)$$

130. To find the *effective* horse power, see "Indicator" (149).

There are other methods which scarcely need mention here, such as—

one-horse power = the evaporation of 1 cubic foot of water per hour (not easily ascertained in practice),

or = the consumption of 33 cubic feet per minute of low-pressure steam;

and some divide the area of the piston by 16, 21, 22, &c.

N.B. Horse power varies as the cube of the speed.

TEMPERATURE OF STEAM.

131. What is commonly called *latent heat* may be thus explained.

If we take a piece of lead of the ordinary temperature, and strike it a few heavy blows with a hammer, we shall find its temperature much increased merely from the concussion.

If we take sulphuric acid and water of ordinary temperature, and mix them in any proportion, the temperature of the mixture will be raised, so much so, that when the mixture is of 4 parts acid and 1 of water, the quantity of heat evolved exceeds considerably that of boiling water, as the mixture will be found to have reached to the temperature of 300° Fahr., or 88° above boiling water.

If we mix pounded ice at the temperature of 32° with an equal weight of hot water, say of 172°, the mixture will still remain at the temperature of 32°; therefore the 140° was *latent* or *hidden* in the ice. If, moreover, we fire a heavy shot against an iron-plated ship, even though the shot should not penetrate, a strong flash of vivid light will be suddenly seen, indicating the disturbance of *latent*

heat in the metal;—the striking of a steel with a flint is of the same kind of disturbance upon a small scale.

If we place water, which is at a temperature near the freezing point, in a favourable position to become regularly heated, it will be found that, comparing the absorption of heat per minute with the time it takes to evaporate all the water into steam, the amount of heat so absorbed will be about 950°; and, as such water would have absorbed 180° between the freezing and boiling point at 212°, therefore the sum of these would be 1130°—the quantity of heat required to dissipate water from 32° into steam of the *sensible* temperature (as affecting the thermometer) of 212° only; consequently the 950° is said to be *latent*, or hidden in the steam. It has been, moreover, found that the higher the temperature of the steam the less is the quantity of latent heat; but the sum of the two kinds of heat is said by some to be a constant quantity, or 1130°; others think that the latent heat itself is a constant. In illustration,—steam at 250° is said to contain 218° sensible heat, and 912° latent, in all 1130°; and, again, steam or vapour at 100° has 68° sensible heat, but 1062° latent; in all 1130°. We thus see that a large quantity or amount of heat can be absorbed by substances without its being indicated by the thermometer; hence the accepted term "latent heat."

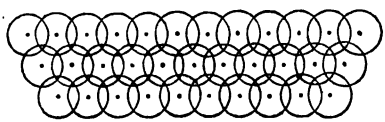
But before we leave this subject it is necessary to consider the circumstances under which so large an amount of latent heat can enter all substances—all ponderable bodies.

132. At p. 120 it was said that steam was a mere mechanical combination of water and heat—(*mechanical* because there is no chemical change to be produced); but as it is an axiom that no two bodies can occupy the same space at the same moment, it is, therefore, natural to infer that if a piece of metal or water can receive a large

accession of heat into its mass, that which is commonly called *conduction* of heat, being somewhat of the nature of what in fluids is called capillary attraction, can only take place among or *between* the minute particles called "atoms," which are *not in absolute contact:* for, otherwise, the above axiom would forbid our admitting the possibility of anything entering a space already absolutely filled with matter.

We shall arrive at some understanding of the nature of substances in general, if we assume that a portion of a substance immensely magnified would appear as in the following diagram.

Fig. 65.



ATOMIC THEORY.

133. Suppose, now, the black dots in the figure to be infinitely small and *indivisible* "atoms" (as they are therefore called) of a substance, say of metal. It is probable that not only metal, but every substance in nature, contains its quantity of electricity; certain it is that there is a mysterious connection between electricity and what is called by chemists "cohesion;" and, as we know that every current of electricity has its current of magnetism at right angles to it, so we are inclined to ask if there may not be a species of *magnetic cohesion* around every atom, reminding us, by its insensibility to the magnet itself, of the relationship of *latent* to sensible heat: in short, is this cohesion a species of latent mag-

netism, *extremely limited in its sphere of operation*, while, for some yet unexplained reasons, capable of extension in some metals, as in iron and cobalt? It is this "limit" with which we have to deal in considering the latent heat in steam, for a supply of heat beyond a certain quantity parts the atoms of water beyond such limit, and the water becoming steam assumes a different form in consequence. There appears to be a state of equilibrium with regard to cohesion, which is favoured by a due supply of heat; for, as it applies to metals, abstraction of heat causes a contraction which renders metals brittle, while an excess of heat causes what we call "*fusion*." An extraordinary illustration of the subject of latent heat has been lately seen in the firing of the Armstrong gun at the iron shield of Captain Coles, as fitted in the Trusty. The immense velocity of the shot striking the iron shield, caused, in some cases, an instantaneous breaking up of the shot into fragments, evidently caused by the suddenness of the abstraction of latent heat, which rendered the metal brittle from having thus lost its heat, the cause of its elasticity. We have here a plausible solution of the question of latent heat; for, while bodies are at a moderate temperature, the spheres of attraction of particles (represented by the circle round each dot in the above figure) overlap each other in every direction, and cohesion or a state of equilibrium exists; but when we admit heat to such mass of metal, it forcibly permeates this mass in all directions, and when this force separates the atoms, so that these imaginary limits of attraction no longer overlap each other, the state called cohesion ceases to exist. Melting or fusion is then the condition of the substance; and it is only mechanical, not chemical, because, on again depriving the molten mass of its heat, it resumes the solid state, and "cools," as the term is,—the atoms recovering their former average cohesive distance. But there is this remarkable circumstance in corroboration,

of not perhaps exactly the above, but of there existing a some such state of things as that above described; for the shipwright knows that in driving a tightly-fitting copper bolt into oak, it cannot be forced beyond a certain power of endurance in the copper, after which the metal has a tendency to "upset" (or thicken in diameter); but let the copper bolt under such circumstances *rest* for a few hours, and it resumes its former dimensions (cools), and the bolt can then more easily be driven home. Workmen tell you that the metal "must have time to recover its elasticity." In the same way that nature, if left to its own operations, will favour the cure of a wound in the flesh, so do nature's laws favour the state of equilibrium in metals referred to. We know that, hammer a piece of iron as we please, if we subject it to a red heat, the thereby lost "elasticity" or state of equilibrium of its particles is restored. If we wish to soften wire, or "anneal" it, we heat it; and elasticity seems therefore to depend on the amount of latent heat the body contains. Liquefaction, then, or melting, seems, as before stated, to be a "mechanical combination" of a substance with heat; as, for instance, ice and heat form water: so that we actually have in steam nothing but the process of liquefaction carried to extreme limits. As a further illustration of this,—if we take 11 cubic inches of water at 32° and raise it to 212°, it will fill 13 inches in measure, and any attempt to heat it beyond this causes evaporation, the continued supply of heat furnishing that which is so essential to the state of steam. And, on the reverse, the 4 ounces of acid and 1 ounce of water already referred to, would not fill a 5-ounce measure, from the large escape of latent heat becoming sensible heat and flying off, and allowing the atoms to lie in closer proximity.

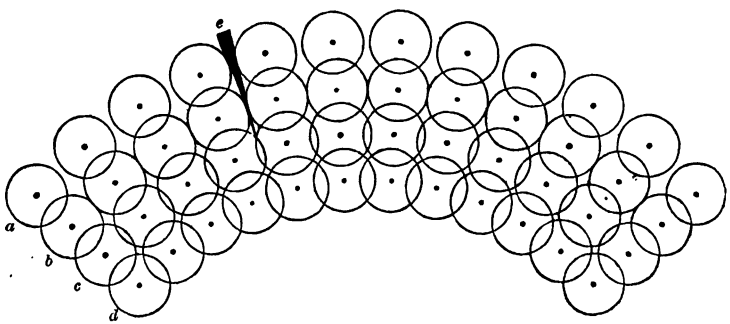
134. It is this power of "affinity," as chemists term the aptitude of one substance to mix with certain others, that facilitates the combinations which take place when

we disturb the atomic equilibrium of bodies by means of heat in particular, and its operations should, therefore, be familiar to the engineer.

FRACTURES IN METAL.

135. The fact above referred to, that a bar of metal contracts or expands with change of temperature, leads us to another circumstance connected with this subject, viz. a consideration of what a fracture in metal really is; for the quality of the iron constitutes the main value of a machine, and such may be *brittle*, or, as the other extreme, *tough*. We know, from sad experience of railway accidents, that a frosty winter endangers metal exposed to heavy strains in low temperatures. It is well to pursue the question of fractures, in order to clearly comprehend the nature of the material on which the safe working of an engine depends. If we bear in mind that "cold" lessens the *elasticity* of iron, we shall be able, from the following diagram, to better understand what we mean by the limit of elasticity.

Fig. 66.



Suppose the black dots of Fig. 66 to represent the highly magnified atoms of a piece of small bar-iron suddenly bent when very cold; while in that condition, the atoms in the outer layer, *a*, would be liable to be drawn beyond their limits of cohesion, when rupture would take place before the atoms would have time to readjust themselves (if we may so speak), while those in *d* would be driven nearer together, at a loss of latent heat, and at a rising of thermometric temperature. We have a striking illustration of the nature of cohesion in the wooden bow of the archer. It is well known that, when desirous of casting an arrow to the greatest distance, a sudden, jerking draw of the string is more effective than a steady draw. Indeed, much more actual power may be exerted in the former case than in the latter, because the elasticity of the wood is merely strained in the one case but destroyed in the other, (the atoms not having time to return to their places before the cohesion is perfectly destroyed). But, as regards metal, additional heat seems to restore elasticity, so long as the limit of cohesive distance has not been exceeded, as would have been the case at *e*, &c., in the above figure, where a fracture in the metal appears from this cause. And this leads us to the subject of

TESTING THE STRENGTH OF THE METAL.

136. Many parts of the engine are subjected during manufacture to the action of testing by the hydraulic press. Now, while the strain is less than about ten tons to the square inch of sectional area, the metal returns to its former state of elasticity, but at and above that, on removing the test strain, a new arrangement of particles seems to have taken place, as a bar then permanently lengthens or "pulls down" (as we say of a rope), and *loses its elasticity*, a serious consideration with the engineer, *unless he take the precaution to re-heat the bar to redness*, in

order to restore it to its former elastic molecular condition. In the "English Cyclopædia of Arts and Sciences," at page 538, the author has fully considered the effect of the injudicious over-testing of iron.

PRESSURE OF STEAM.

137. Pressure, or the "force" of steam, is a question which presents itself to us under various forms, some principles of which will be illustrated. We avail ourselves of two important decisions as regards this question.

138. From "Marriotte's law," as it is called, we find that "when the volume of an elastic fluid is increased, its elasticity or pressure is increased in the same ratio."

139. From Gay Lussac we learn that "an elastic fluid, varying in temperature (its tension being the same), will increase in volume in proportion as it increases in temperature;" and he determines that for every degree of increase of temperature (Fahrenheit) there will be an increase of volume of ·00202 (or very nearly $\frac{1}{500}$th of the volume occupied by the fluid at the temperature of 32°). (This applies to fresh water.)

EXAMPLE.—What is the force of steam in inches of mercury when the temperature of the steam is 300° Fahr.?

By Watt's well-known formula,

$$\frac{\text{the temperature} + 100^{\circ}}{177} = (\text{the force})^{6}$$

$$\text{or, in this case, } \frac{300 + 100}{177} = (\text{the force})^{6}$$

By logarithms, (74)

log of 400 = 2·602060
log of 177 = 2·247973
0·354087
6 = index of power
Answer, force or pressure 133·2 in. = 2·124522 = sixth power.

And to find the temperature of steam at a certain pressure, we have

The temperature + 100° = $\sqrt[6]{}$ pressure in inches × 177

By logarithms,

log of pressure 133·2 in. = $\frac{2\cdot124504}{6}$ = ·354084, the 6th root

log of 177 = 2·247973

2·602057 = 400

− 100

300 the answer.

To find the space occupied by a cubic foot of water when converted into steam, we must consequently have the temperature. Suppose, therefore, the given force of steam to be three atmospheres (119), required its temperature and the space it occupies?

To find temperature (as above),

30 in. of mer. in 1 atmosphere

log of 90 = $\frac{1\cdot954243}{6}$ = ·325707, the 6th root of $\overline{90}^{3}$ inches in 3 atmospheres

log of 177 = 2·247973

2·573680 = 374·7

− 100

274·7, the temperature.

To find the space,

$$\frac{(\text{the temperature} + 459) \times 76\cdot5}{\text{force of steam in in. of mercury}} = \frac{(274\cdot7 + 459) \times 76\cdot5}{30 \times 3} = 624 \text{ cubic feet.}$$

To find the increase of volume at any temperature, say at 300° Fahr.

300 − 32 = 268

∴ 268 × ·00202 = ·536, the increase of volume according to Gay Lussac's rule above.

N.B. The numbers 459 and ·00202, &c., are called *constants* when they apply to examples in general. The source of these will be properly explained in the Author's intended sequel to this work.

EXPANSION.

140. Working a steam engine by expansion is the taking advantage of the elasticity of steam as a means of economy. The mode of application is by shutting off the steam from the cylinder before the full length of the stroke of the piston has been made. The power of the steam introduced before such cutting off is allowed to develope itself during the remainder of the stroke. When the steam during its entering the cylinder continues of a uniform density, the velocity of the piston is continually increasing until the end of the stroke, but in such case the sudden reversal of the direction of the steam causes a shock and great strain upon the machinery; and this is not only prevented by working expansively, but an absolute saving of steam, and therefore of fuel, is effected. The economy consists in a certain quantity of steam being made to produce a greater amount of power, although at the loss of some speed; in other words, a certain quantity of coal is made to drive a ship along a greater distance.

141. Various questions in practice arise as to working expansively, but the following will furnish ample instructions to the practical engineer: —

If we let a = length of stroke (in inches)	then $a = \frac{bc}{d}$	
b = the part of stroke cut off before expansion (in inches)	"	$b = \frac{da}{c} = \frac{hi}{c}$
c = whole pressure in boiler	"	$c = \frac{ad}{b} = \frac{hi}{b}$
d = terminal pressure in cylinder	"	$d = \frac{bc}{a} = \frac{b \times (h+15)}{a}$
e = mean pressure in cylinder	"	$e = c - d = \frac{a}{bc} - d$
h = load in lbs. on sq. inch of piston	"	$h = \frac{bc}{i}$

i = point of stroke when velocity of piston is greatest — then $i = \frac{bc}{k}$

p = expansion — „ $p = \frac{a}{b}$

The method of using these is the following.

Suppose we have given the pressure in the boiler 40 lbs., length of stroke 10 feet, and steam cut off at 3 feet, required the terminal pressure; then (see previous page)

$c = 40$ lbs.
$a = 120$ inches
$b = 36$ „
} to find the terminal pressure d.

Reference to the last page for an expression of the value of d would give the equation

$$d = \frac{bc}{a} \text{ or } d = \frac{36 \times 40}{120} = 12 \text{ lbs.}$$

And again —

Given pressure 50 lbs., terminal pressure 20 lbs., stroke 3 ft. 10 in., required to know at what point the expansion commenced.

Then, by the symbols at the previous page,

Pressure $= c = 50$ lbs.
Terminal pressure $= d = 20$ lbs.
Stroke $= a = 46$ ins.
} to find point of "cut-off" $= b$

By formula, $b = \frac{da}{c} = \frac{20 \times 46}{50} = 18{\cdot}4$ in. $= 1$ ft. $6{\cdot}4$ in.

N.B. If the student have any difficulty in applying these formulæ, he should go back to page 9 for information.

142. If it be required to know at what point of the stroke it is most beneficial to cut off steam, such will depend entirely upon the force of steam in the boiler, as estimated in inches of mercury; for this force divided by 10 gives an easy approximation to the maximum power of work; thus, suppose the force of steam in the boiler to be 42 inches, then $\frac{42}{10} = 4{\cdot}2$, or $\frac{1}{4{\cdot}2}$ or $\frac{10}{42} = \frac{5}{21}$ the length of the stroke.

N.B. The number 10 is the estimate by Tredgold for the friction of the machinery, and the force of the vapour in the condenser as equal to 5lbs. per square inch, or 10 inches of mercury (119).

CONSUMPTION OF COAL AND SPEED OF VESSEL.

143. The consumption of coal varies as the distance multiplied into the square of the speed, or as the cube of the speed itself.

For example:—Suppose a ship uses 200 tons of coal in a voyage of 500 miles, and steams at 10 knots an hour, what quantity would take her 1000 miles at 6 knots an hour?

$$(500 \times 10^2) : (1000 \times 6^2) :: 200 : x$$

$$\text{And } x = \frac{1000 \times 6^2 \times 200}{500 \times 10^2} = \frac{36000 \times 200}{50000} = \frac{7200000}{50000}$$

$$= \frac{720}{5} = 144 \text{ tons.}$$

And, as another example, steaming at 11 knots an hour I used 4 tons of coal: how much should I have used in the same time had I steamed at 9 knots?

$$\text{Thus } 11^3 : 9^3 :: 4 : x$$

$$\text{And } x = \frac{9^3 \times 4}{11^3} = \frac{2916}{1331} = 1{\cdot}44 \text{ tons.}$$

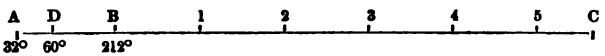
144. It has been found that in steaming against a current, the most economical velocity for the ship is to proceed 50 per cent. faster than such current; that is, supposing the current to run at 4 miles per hour, the ship should steam at 6 miles.

All experiments, therefore, show that a high rate of speed in a vessel can only be attained by greatly increased expense of fuel.

FEED-WATER.

145. Admitting that it takes $5\frac{1}{2}$ times as long to evaporate water, after being raised to boiling point, as it does to raise it from 32° to the boiling point (the heat continuing of uniform intensity), let, for example, 1 hour be the time required to raise water from 32° to 212°, then it will take $6\frac{1}{2}$ hours to boil and evaporate it.

Let one ton of coal be consumed in boiling and evaporating a quantity of water at 32°: required the saving of coal per cent. by using feed-water at various temperatures.



In the above figure, let AB represent lineally the time it takes to raise water from 32° to 212°, and let BC (=5·5 times such period) be the time required to evaporate. Then the whole line AC will equal 6·5 hours; or it will represent the unit of consumption of coal in the period of $6\frac{1}{2}$ hours.

Now, suppose I want to know what saving I effect in using feed-water at 60° instead of at 32°, I see by the figure (where the point A represents water at 32°, and the point B 212°, and I make the point D accordingly represent 60°) that,

AC : 1 ton :: DC : x tons (the fraction that DC is of AC),

Then $x = \frac{DC}{AC} = \frac{(DB+BC)}{AC} = \frac{(180-28)+(180\times 5{\cdot}5)}{180\times 6{\cdot}5} =$

$\frac{152+990}{1170} = \frac{1142}{1170} = {\cdot}976$ of a ton, the consumption; and

$1{\cdot}000 - {\cdot}976 = {\cdot}024$, the saving, equal to about 1 ton in 41.

146. From this we get the following table:—

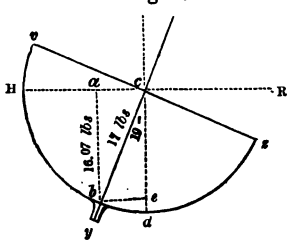
Temp. of feed-water, Fahr.	Quantity of coal required.	Saving of coal.
40°	·993	·007
50	·985	·015
60	·976	·024
70	·968	·032
80	·959	·041
90	·950	·050
100	·942	·058
110	·933	·067
120	·925	·075
130	·916	·084
140	·908	·092
150	·899	·101
160	·890	·110

HEEL OF THE SHIP.

147. The power of the steam when blowing off water from the boiler, depends on the immersion of the aperture used for that purpose in the ship's bottom, or on its distance below the surface of the water.

The following figure will explain this.

Fig. 67.



Let HR represent the horizontal line, and *vyz* the ship's hull inclining or "heeling" 19°, as shown at *yce*. It is evident that if we let *cb* represent the load in the boiler, or say 17 lbs. per square inch, *ab* or *ce* will represent the force the steam will exert in blowing off; and it is plain that, *cb* being the radius of the circle, *ce*

will be as the cosine of the angle bce (for it is as the sine of its complement cbe). But we are in reality dealing with a question in the resolution of forces in which bc is the resultant of the two component forces ac and ce, and we wish to find ce, and do it thus (107):—

By trigonometry:—Rad cb : cos 19° :: 17 lbs : x lbs.

$$(5) \quad \therefore x = \frac{\cos 19° \times 17}{\text{rad}}$$

Or, by logarithms:—

log cos 19°	9·975670
log of 17	1·230449
	1·206119 = ce = 16·07 lbs.

Had we desired to find the angle of inclination, having the two forces given, it would appear that—

$$\text{Rad} : \cos x :: 17 : 16{\cdot}07$$

$$\text{Or, } \cos x = \frac{\text{rad} \times 16{\cdot}07}{17}$$

Or, by logarithms:—

radius	10·000000
log 16·07	1·206119
	11·206119
log 17	1·230449
	9·975670 = cos ∠ bcd = 19°

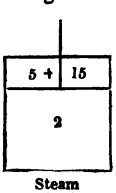
INDICATOR.

148. The steam engine has already (115) been compared to a breathing animal, and the importance of the cylinder rendered apparent, as constituting the lungs or heart of the whole machine, so that an "unhealthy" condition of the cylinder is inconsistent with its accurate working. So deeply was James Watt impressed with the

necessity of becoming acquainted with the *internal condition* of the cylinder during the motion of the piston, that, as the physician avails himself of the principle of auscultation for examining the air-chambers of a phthisical patient, so did Watt construct an instrument by means of which the steam passages of the engine might be in like manner examined. This elegant little instrument he called an indicator. It is unnecessary in this work to give extensive details of the indicator, because a work of great utility, published by Main and Brown, gives every description of it; but a few hints may be acceptable to those who neither have that work nor a knowledge of the instrument.

The indicator is a miniature cylinder, connected with that of the engine in such a manner that when steam in the latter acts upon its piston a like effect is produced on the piston of the indicator; to which is attached a pencil and paper moved by the machinery, so that a correct register is obtained of the *actual power* at work in the engine. And the area of the figure so traced is made to correspond with pounds weight of pressure. A certain zero line, called the atmospheric line, is drawn on the paper, from which are measured both steam and amount of condensation (118). The "working off a diagram," as the operation is called, is performed as follows:—but let the accompanying figure first assist in explaining the term "effective pressure."

Fig. 68.



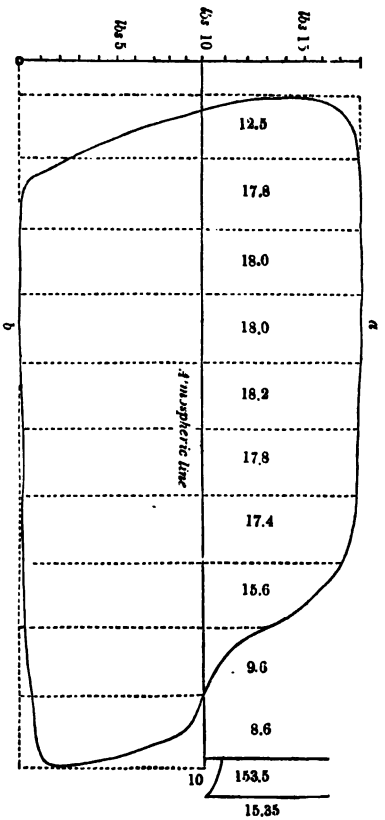
Steam Cylinder.

If the pressure of the steam on one side of the piston be 5 lbs. per square inch of area above the atmospheric pressure (which is 15 lbs.), then $5+15$ lbs. $=20$; and if on the other side the piston there be 2 lbs. per square inch pressure (from uncondensed steam, &c.), then $20-2=$ 18 lbs. = the effective pressure; or the condenser would have exhausted steam up to 13 lbs. per

inch (complete exhaustion being 15 lbs. per inch), or we should have a "26-inch vacuum" (30 inches of mercury being a *complete* vacuum) (119).

149. In the following copy of an actual diagram, the horizontal line represents the atmospheric line; a graduated scale is on the left, which measures the number of pounds pressure upon every square inch of the piston surface, as previously ascertained by the maker. Nine ordinates to the atmospheric line are drawn at equal distances, thus dividing the space in length into 10 equal parts; and, as an example, the length ab indicates that in the interval, while the parts of the curves at a and at b were being traced by the indicator, there was a pressure in the cylinder of 18 lbs. to the square inch; the sum of all these areas divided by 10 gives the average mean pressure of steam and vacuum in the cylinder.

Fig. 69.



Now, we have the means of finding the effective horse-power, as under—

$$\text{The effective horse power of cylinder (irrespective of friction)} = \frac{\text{area of cylin.} \times \text{pressure of steam as found by indicator} \times \text{speed of piston}}{33000}$$

$$\text{or} = \frac{(\text{dia. of cy.} \times \cdot 7854) \times \text{pr.} \times (2 \text{ strokes} \times \text{no of revolutions})}{33000}$$

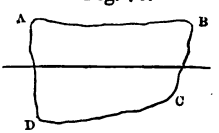
This calculation gives horse power less than the truth, from the friction of the parts of the indicator. Other calculations may be made from the diagram, such as the quantity of water evaporated; but several things, such as loss of heat by the steam in its passage, &c., prevent desirable accuracy; otherwise the volume of steam entering the cylinder at every stroke of the piston, being of an ascertained temperature, would give the true consumption of steam, and hence the quantity of water evaporated.

150. In order to give some notion of the value of the indicator, a few miniature diagrams are added, together with some practical hints as to the important information obtainable from them.

The actual diagrams are generally about $3\frac{1}{2}$ inches laterally.

A is the "steam corner" or commencement of stroke, and by being very angular shows that the valve is too forward at the beginning.

Fig. 70.



AB being nearly parallel to atmospheric line, shows that there was no expansion, but supply of steam to the end of stroke, with pretty equal pressure.

C is the "exhaust corner," and being rounded off shows that the exhaust valve opens too late; therefore power is lost, because steam does not get into the condenser soon enough.

D is also an exhaust corner, or the eduction corner; it is too square, and shows that the valve opens too late.

N.B. In all these diagrams the horizontal line is the atmospheric line. The fulness of the part below this line shows that a good vacuum was obtained; the depression at B shows that the steam was cut off there at about two-thirds the whole stroke of the piston (as measured by xy); that from B to C the steam was acting expansively.

(Fig. 71.) A fair average diagram, showing, however, at A, that, if not caused by the friction of the piston of the indicator itself, there was too sudden a rush of steam, which is trying to the machinery from its abruptness.

Fig. 71.

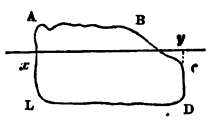
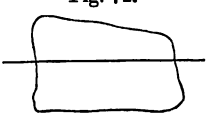


Fig. 72.



(Fig. 72.) This engine has large steam ways, and there is a very large quantity of steam to be condensed at the end of the stroke. There needs lap to valve; this would save steam.

(Fig. 73.) Steam cut off at half stroke, at the end of which there was (as shown at a) some steam to condense; b shows that there was a good vacuum.

Fig. 73.

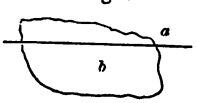
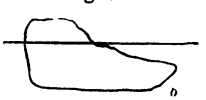


Fig. 74.



(Fig. 74.) Steam cut off at one-third stroke. Exhaust opens too soon (loss of power); b is too much rounded.

(Fig. 75.) Steam ways very small, as shown by smallness of *a*, the steam side; the vacuum side is bad, for much steam is uncondensed; it causes also noise in cylinder from passages being too small.

Exhaust valve opens too late, and steam cannot leave the cylinder at proper time (see at *b*); wants "lead" on the exhaust side. Hence waste of fuel and great loss of power.

Fig. 75.

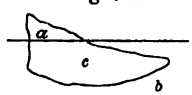
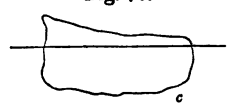


Fig. 76.



(Fig. 76.) Slide valve requires more lead and more lap. Exhaust opens too late and closes too late. Steam ways too small altogether. Vacuum is pretty good, but too rounded at the exhaust corner.

(Fig. 77.) *a* shows, by being too much rounded and hollowed, that the valve has too much lead; the cut-off at *b* shows expansion; it is, however, a pretty good diagram; exhaust opens rather early.

Fig. 77.

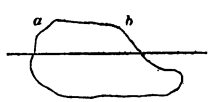
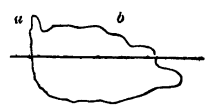


Fig. 78.



(Fig. 78.) *a* shows that the valve has no lead, and that the pressure of steam strikes the piston with some force, to the great strain of the engine by its suddenness; the part *b* shows that the cut-off was not good; the steam was "wire-drawn," entering after the "cut-off," and indeed the wavy lines of the figure indicate great derangement of the valves generally.

(Fig. 79.) The steam too late, and the exhaust too soon. Valves out of order. The rounded corner at *a* is called cushioning (*i. e.* valve shuts before the end of the stroke).

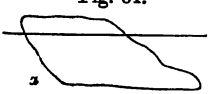
Fig. 79.

Fig. 80.

(Fig. 80.) A very fair diagram; steam cut off at first grade (H.M.S. Pylades); *a* was caused by stiffness in the indicator (*i. e.* friction of the indicator piston).

(Fig. 81.) H.M.S. Pylades. Wind on the quarter, all plain sail set, and studding sails on one side. *a* shows too much "cushioning."

Fig. 81.



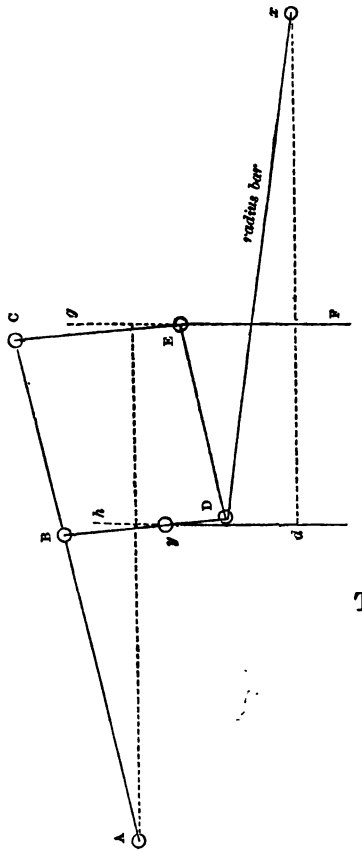
N.B. The change of $\frac{1}{8}$ in. in the valve on this occasion set it all right. Cushioning is the too early closing of the eduction valve before the end of the return stroke, and thereby steam is shut in and compressed by the piston.

PARALLEL MOTION AND LINK MOTION.

151. At page 122, it has been shown that motion in a steam engine is communicated by means of the rise and fall of the piston, the rod of which, in a fixed cylinder, must necessarily move in a vertical direction, if we would avoid loss by excessive friction; that this rise and fall moves the end of an arm of a lever or "beam" working on a fixed centre, and therefore describing a curve instead of a straight line.

An arrangement called a "parallel motion" is used to preserve the piston rod in its vertical position throughout the stroke. It is not, however, a precisely vertical line, but the deviation from it is not perceptible in ordinary practice.

Fig. 82.



152. In the following figure, suppose it were desirable to fix a "radius bar" to the parallelogram DBCED: in order to accomplish this we should call—

	feet
A C the ½ beam moving on its centre say . .	= 6½
D E = B C the parallel bar	= 2½
C E the main link . . .	= 2
B D the link proper . .	= 2
Dx the radius bar.	

$$\text{Then } Dx = \frac{AB^2}{BC} = \frac{16}{2\frac{1}{2}} = 6{\cdot}4$$

The point x would be fixed when the parallel bar DE coincided with dx (a horizontal line, or a line at right angles to the line of stroke), and when CEF formed a continued right line, the parallelogram being at such moment rectangular, and the effect of the parallel motion is to make the point E traverse upon the line Fg.

153. In the above figure, the link, properly so called, is the bar BD, because it is the regulator of the whole motion, inasmuch as it connects the two arms AB and Dx. Advantage is therefore taken to use the vertical motion of a point in BD, for the guidance of, for instance, the piston of the air pump; and as *the segments of the link are inversely proportional to the lengths of the arms or radii*, we find the point y as under (from the last figure).

$$(AB+Dx) : AB :: BD : Dy$$
$$\text{or } (4+6{\cdot}4) : 4 :: 2 : Dy$$
$$\text{or } Dy \times 10{\cdot}4 = 8$$
$$\text{or } Dy = \frac{8}{10{\cdot}4} = {\cdot}77$$

The air-pump piston would therefore traverse vertically also on the line Dh. An elegant application of the link motion has been made by Mr. Robert Stephenson (called Stephenson's link motion). It is a contrivance for regulating the slide-valve gear for reversing engines, and is also applied to regulate and vary the amount of expansion, but it would be difficult without a working model to convey a correct impression of its action. It is formed by connecting two eccentric rods by a link, a slot in which receives a stud on the slide-valve rod.

AIR PUMP

154. Is used to remove the condensed water, air, &c., which collects in the condenser, passing it onward through the delivery valves into the hot well, from which the supply of water is derived to replenish the boiler; the temperature of such condensed water and other contrivances being available in preventing the boiler being supplied with cold water, as such would involve an unnecessary loss of fuel.

Questions on the air pump which arise in practice, are mostly mere considerations of area and volume. But inasmuch as air pumps are required to act at different depths beneath the sea-surface level, where the weight of external water counteracts the atmospheric pressure, according to such depths, the following calculations are often required by the engineer — in estimating the power of blowing off, for example.

Let L = the load on the air-pump bucket in pounds.
f = the vertical depth in feet of the bucket beneath the sea-level.
P = the pressure of atmosphere at the sea-level, or 14·75 lbs. per square inch.
p = the pressure of steam in pounds in condenser.
d = the downward pressure of the water in pounds per square inch.

A cubic foot of sea water weighing 64 lbs., and 34 inches = a column of water at atmospheric pressure (121),

$\therefore \frac{64}{144} = \frac{4}{9}$ will equal the pressure of 1 lb. of water on a square inch.

Then $P-(\frac{4}{9} \times f) = d$
and $d-p = L$
and also $P-(\frac{4}{9} \times f)-p = L$

And again,

$$34 : 14{\cdot}75 :: f : d$$

$$\text{or } d = \left(\frac{14{\cdot}75 \times f}{34}\right)$$

$$\text{and } L = \left(\frac{14{\cdot}75 \times f}{34}\right) + (14{\cdot}75 - p) \times \text{area of bucket.}$$

MERCURIAL GAUGE.

155. The barometer gauge and the syphon gauge are of this kind; but the former is in most general use in Her Majesty's service, and Bourdon's spring gauge in the merchant service. The use of the gauge is to ascertain the amount of vacuum in the condenser; hence it is called the *vacuum gauge.*

The barometer gauge is a metal tube connected with the condenser. It passes up through a metal cup of mercury, is open at the top, over which is inverted a glass tube sealed at the upper end, and standing in the cup of mercury. When the pressure upon the mercury outside the tube equals the pressure in the condenser, the level of the mercury is the zero of the attached scale of inches; but as soon as a vacuum commences, the air within the glass and metal tube becomes "rarefied," and, the pressure being thus relieved, the mercury is drawn above the level within the glass tube, and the amount of its rise indicates the inches of diminished pressure or "vacuum." The suddenness of condensation often causes difficulty in reading off, from the rapid motion of the mercury when the pressure is high.

The syphon gauge is formed of two lengths of tubing, connected at the lower end by a tube bent into a semicircle, the upper end of one tube being open to the atmosphere, and the other cemented air-tight into another bent tube, the lower end of which is connected with the condenser. The tubes contain mercury; and exhaustion in the condenser causes the rising of the mercury in the one tube and its falling in the other: the difference in the levels indicates the amount of diminished pressure, or vacuum. The main difficulty in graduating the syphon, is the difference which usually exists in the

sectional area of the two tubes, since few tubes are, indeed, perfect cylinders. A number of interesting questions arise under these circumstances, of the following nature: —

Let the fall of mercury in one leg = 11
Let the rise and fall in the other = 6
Then the diameter will be as $\sqrt{\frac{11}{6}}$
or the fall 11 : the rise 6 :: $\sqrt{11}$: $\sqrt{6}$

And again,—

The areas are as the fall 11 : the rise 6 :: $\overset{\text{Area}}{11}$: $\overset{\text{Area}}{6}$

As a general expression—

Let a = the area of the leg open to air
b = the area of the leg open to condenser or boiler
p = the pressure of steam per inch
w = the weight of cubic inch of mercury
h = the height of index when steam blows off.

$$\text{Then } \frac{a}{b} = \frac{w\,h}{p} - 1$$

Bourdon's spring gauge seems to satisfy those who use it, and avoids necessity for these considerations. Its appearance need not be given here, as it is well known.

CYLINDER.

156. Numerous questions upon the cylinder occur in practice, but they are principally those of areas and solids, or comparisons and ratios of areas and solids. The use and action of the cylinder has already been explained (116).

THE CRANK.

157. Linear motion is converted into circular motion by means of a crank (116). In the steam engine the

piston acts upon a connecting rod attached to a crank which turns the main shaft like the handle of a windlass. In the following figure, the piston is supposed to be acting in the direction $x\ y$. x being the position of the piston-head at the commencement of the upward stroke, n being the connecting rod, and m the crank, then y would be the shaft centre.

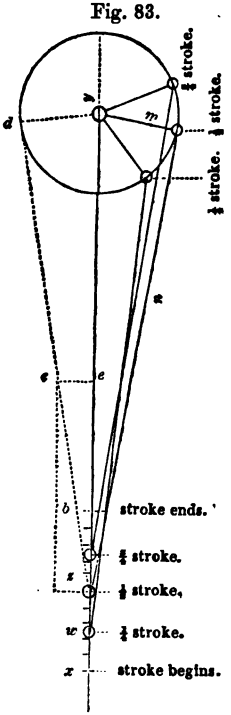
It is evident that at two parts of the revolution of the crank (that is, when the axis of the crank is in the same line with the axis of the piston-rod) no circular motion could be produced, as there would be a dead upward thrust on the shaft centre; these are called the "dead points;" and to obviate this inconvenience, recourse is had in land engines to a *fly-wheel*, the object of which is to carry the crank beyond these dead points, by the impetus previously given to the wheel, which tends thus to equalise the action of the machine. Marine engines are generally in pairs, and, the cranks being set at different angles on the shaft, the one crank carries the other beyond the dead point.

On examining the following diagram, it will be seen that the half stroke is not when the crank-pin has traversed one half the circle (as the beginner is apt to imagine), but when the piston-head itself has traversed the half stroke, and the figure shows also that the crank travels at different velocities. The crank-pin could be at the half of the semicircle only when the power is applied in a direction throughout parallel to the line of action of the cylinder; a state of things incompatible with the construction of the steam engine.

158. Another erroneous impression springs from the inequality of the motion of the crank, viz., that it indicates a loss of power; but it is not so. Professor Woolhouse and others have demonstrated that not only is power neither lost nor gained in the use of the crank, but that such applies equally to all possible mechanical combinations.

Loss, therefore, arises from friction only; but the inequality of the motion of the crank is diminished in proportion as we lengthen the connecting rod with regard to the crank.

Fig. 83.



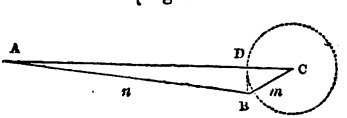
Calculations upon the motion of the crank are of the following nature, depending chiefly on trigonometry.

Suppose, for example, a force of one ton to be acting on the piston in a vertical direction: required to know the amount of thrust on the shaft centre, the position of the crank at any portion of the stroke, the lateral thrust on the piston-head, and the angle made by the connecting rod and the piston — the crank being, say, 12 inches long, and the connecting rod 6 feet.

Let n, the connecting rod = 6 ft.
m, the crank = 12 in.

159. 1. To find the angle between piston and connecting rod (say at ¼ stroke of the engine). We require the angles of the triangle ABC, of which the side AB is 6, BC is 1, and AC (Fig. 84), say, 6·5.

Fig. 84.



In this three sides are given (62). Draw a perpendicular from B on AC.

$$\therefore\ AC : AB+BC :: AB-BC : AD-DC.$$

$$(5)\ \text{or}\ (AD-DC)=\frac{(AB+BC)\times(AB-BC)}{AC}$$

or, by logarithms (62):—

As side A C 6·5	ar co	9·187087		
Is to A B + B C 6 + 1 = 7	.	0·845098		
So is A B − B C 5	.	0·698970		
To diff. of segments	.	9·731155 = 2)	5·384	2)6·5
			2·692	3·25
				2·692
			AD =	0·942

To find ∠A (46):—

As A B 6	ar co	0·221849
Is to rad		10·
So is A D 5·942		0·773933
		0·995782 = 7° 58′ = ∠A.

160. 2. To find the upward and lateral thrusts. (Fig. 83.)

Suppose the crank to be at *d*, *dz* being the connecting rod, and the angle *dzy* the angle at the piston-head. A reference to forces (107) will show that if the thrust in the direction of the connecting rod be as given above—one ton,—we can resolve it into its required components by drawing vertical and horizontal lines—completing the "parallelogram of forces," and that *zc* will be the resultant; then, if *zc* be made equal to 2240 lbs (one ton), *ce* will measure on the same scale of equal parts 354·8, being the lateral thrust, and *ze* the vertical thrust required, or 2212 lbs. Take the ∠*cze* as, say, = 9°, then, by trigonometry (45):—

$$\text{rad} : 2240 :: \sin 9^\circ : \text{side } ce$$
$$:: \cos 9^\circ : \text{side } ze$$

$$\text{or side } ce = \frac{2240 \times \sin 9^\circ}{\text{rad}}$$

$$\text{and side } ze = \frac{2240 \times \cos 9^\circ}{\text{rad}}$$

By logarithms:—

Log 2240 . .	3·550248	. . . . .	3·550248
Sin 9° . . .	9·194332	Cos 9° . . .	9·994620
ce = 354·8	= 2·744580	ze = 2212	= 3·544868

BLOWING-THROUGH VALVE AND SNIFTING VALVE.

161. Before starting an engine, the blow-through valve is used to blow steam through the engine to remove all condensed water or air, which would otherwise impair the vacuum. This is done by the steam in the cylinder jacket being permitted, by means of the blow-through valve, to enter the condenser without entering the cylinder; and its escape into the air is through what is called a "tail" valve, or more commonly a "snifting" valve. When, therefore, steam issues from this snifting valve, the condenser contains steam only, and the engineer may start the engine. (N.B. A cylinder jacket is an outer case upon a cylinder to prevent loss of heat. Questions on this subject differ in no respect from those of areas and pressures already explained.)

BLOWING-OUT VALVE.

162. Necessity for occasionally "blowing out" the water from the boiler arises from the impurities contained

in the water, as the process of evaporation leaves them in the boiler. Steam consists of pure water alone, hence its formation causes accumulations in the bottom of the boiler, which would soon prove its destruction; for, such solid matter being a bad conductor of heat, the furnace would soon overheat the plates of the boiler, where covered, internally with such deposits, and such becoming red-hot, the danger of explosion and rupture of the plates would be the consequence. So that the blowing out the boiler is the ejection of this accumulation of "brine" before it becomes heated into a solid form. There is at the bottom of boilers a "blowing-out" cock, attached to a pipe which leads through the ship's side into the sea; but not before it has been allowed to pass in a pipe through the hot well, to increase the temperature of the replenishing or feed water (145) before pumping it into the boiler to take the place of the "brine" which has become unfit for use. Generally, every stroke of the piston pumps out a certain portion of such brine, and pumps into the boiler a corresponding quantity of feed-water.

Under the word "air-pump" (154) it was noticed that the position of the water level in the boiler often differs from that outside the ship; this would affect considerably the process of blowing out if the boiler be placed low in the ship; hence calculations of the following nature are at times requisite. Under "Atmospheric Pressure" (119) it has been noticed and explained that atmospheric pressure, at 14·75 lbs. of mercury, sustains a column of water in a vertical tube to the height of about 34 feet (121). We have, then, this analogy: —

$$14{\cdot}75 : 34 :: \left\{\begin{array}{c}\text{any given pressure}\\ \text{of mercury.}\end{array}\right\} : \begin{array}{l}\text{its corresponding}\\ \text{vertical column.}\end{array}$$

This will solve most questions which are likely to arise on this subject. But as regards the saline matter which is held

in solution in the "brine" of the boiler, it must be remembered that sea water in its ordinary state contains about $\frac{1}{33}$ of such impurities; for 1 cubic foot of sea water weighs 64·3 lbs., and $\frac{64{\cdot}3}{33} = 1{\cdot}89$, and $1{\cdot}89 + 62{\cdot}425$ (weight of a cubic foot of fresh water) $= 64{\cdot}315$ (= cubic foot of sea water).

And if B represent the quantity of water blown out in any given time,

and b = the quantity of water lost from the boiler by conversion into steam in the same time,

then $B + b$ will = the quantity of water used in the boiler in that time,

and $\frac{B+b}{33}$ = the quantity of solid matter formed in the boiler in the same time.

And if $\frac{3}{33}$ = the increased strength or accumulation of saline matter blown out in such time,

then $\frac{3\,B}{33}$ will = the quantity (at such strength) blown out in such time.

$$\therefore \frac{B+b}{33} = \frac{3\,B}{33}$$

$$\text{or } B+b = 3\,B$$

$$\text{or } B = \frac{b}{2}$$

Therefore, the strength of the brine being $\frac{3}{33}$, the quantity blown out would be equal to half the quantity evaporated.

SEAWARD'S SLIDES

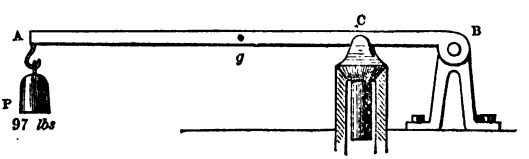
163. Are merely an appliance in Seaward's engines to admit the steam into and out of the cylinder. Various other slides (such as the D slide, &c.) are used for the same purpose, but their general use is to regulate the admission of steam by opening the "ports" or doors of the cylinder, and requires no especial mode of calculation beyond ordinary mensuration.

SAFETY VALVES.

164. Every boiler is tested to bear a certain amount of pressure per square inch of internal surface, but only a portion of this pressure is allowed in practice. To prevent explosions from the too great pressure in boilers, an apparatus is attached to the steam chest, which consists in principle of an escape tube, which is loaded with a weight heavy enough to confine steam and prevent its waste, but not heavy enough to allow a dangerous quantity of steam to accumulate in the boiler. This apparatus is therefore called a "safety valve," and is by law inaccessible to the engine-driver; who, however, has control over another safety valve (there are frequently more than one) for the purpose of letting out surplus steam, &c.

The weight which closes the valve is usually kept in its place by a lever of the third kind. The following figure is given in illustration.

Fig. 85.



As an example of the nature of the questions which refer to the lever of a safety valve, we will suppose—

The lever to be 20 inches long, and weighing 6 lbs.

A weight of 97 lbs. is attached to the end of the lever at A.

The weight of the safety valve and its load to be 100 lbs.

How far must the valve centre lie from the fulcrum B so as to resist a pressure of steam of 525 lbs. on the valve face?

Let $P = 97$ lbs.,

g = centre of gravity of the bar itself,

$w = 6$ lbs. (weight of lever). Then, by (103),

$$B\ C = \frac{(P \times AB) + (w \times Bg)}{100 + 525}$$

$$= \frac{(97 \times 20) + (6 \times 10)}{625} = \frac{1940 + 60}{625} =$$

$$\frac{2000}{625} = 3{\cdot}2 \text{ in.}$$

And, of course, to find the weight necessary to hang at the end of the bar to produce this effect, having the distance of the point of pressure given, we should have from the above

$$\frac{(x \times 20) + (6 \times 10)}{625} = 3{\cdot}2$$

$$x = \frac{(625 \times 3{\cdot}2) - (6 \times 10)}{20} = \frac{2000 - 60}{20} = \frac{1940}{20} = 97 \text{ lbs.}$$

But all questions on levers should be referred to the formula for those of the first kind, viz., that the moment of power must equal the moment of weight. Nor does there being weight in the bar prevent this, if we resolve

the above question, for instance, which is a lever of the third kind, into one of the first kind (98). See the following figure.

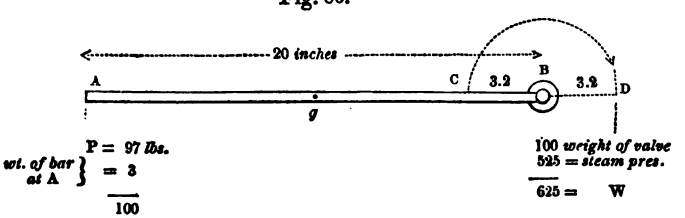
165. *First*, taking into consideration the weight of the bar itself;—under "Gravity" (94) we show that g is in this instance the point at which the weight of the bar ($=6$ lbs.) acts with regard to the fulcrum, but, taking the previous example, 20 in. : 10 in. :: 6 lbs. : 3 lbs.; therefore, whether we have a weight of 6 lbs. at g, or 3 lbs. at A, the same power is produced.

166. And, *secondly*, if we resolve the third kind into the first, and, by reversing the direction of the fulcrum at B, suppose the pressure to be a weight hanging beyond B, or at D, at a distance equal to BC, we shall have in the same example

$$625 \times 3{\cdot}2 = (97+3) \times 20$$
$$2000 \quad = \quad 2000$$

the "3" used in the equation being the weight of the bar if transferred to the same point as the weight 97 hangs from.

Fig. 86.



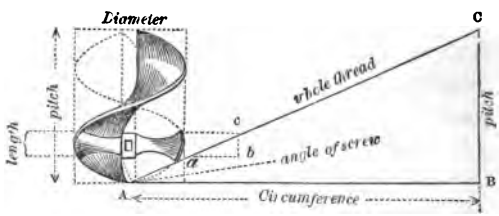
THE SCREW PROPELLER.

167. Not only in Her Majesty's service, but in the mercantile marine, the screw propeller offers such advantages

over the paddle, that the details of its principles are of the highest importance. The question of the superiority of one form of screw over another need not occupy our *present* attention, but the properties of the screw propeller and modes of calculation of screw machinery will be briefly put before the student, in such form as will enable him to enter upon further investigation without difficulty.

The screw propeller in common use is merely a section of the screw at right angles to the axis; sometimes of a two-threaded screw; at others of a three-threaded, having two or three blades accordingly. For simplicity of illustration we adopt the former. In the following figure will be seen the origin of all the terms used in speaking of or writing upon this subject.

Fig. 87.



It represents *one whole turn* of a deep two-threaded screw (although only one thread is shaded), of such proportions as would furnish a section of what is in common use, say pitch about 16 feet, diameter 12 feet, and length about 2 feet 8 inches. It is usual to consider the thread of a screw as the hypotenuse of a triangle whose base equals the circumference of the cylinder from which it is cut; the position of the thread on such cylinder being caused by wrapping the triangle round the cylinder, the perpendicular being the "pitch" or axial length of a whole turn of the thread. Such triangle being supposed to be unrolled

from the screw in the figure, would appear on a plane as at ABC, of which AB is the diameter × 3·1416. In the smaller and "similar" triangle (26), abc, bc = the length, and ac the length of the thread of the screw-blade itself, while AC = the whole length of the thread of one whole turn of the cylinder.

168. Therefore, by (19) —

$$\tan \angle A = \frac{BC}{AB} \text{ or } = \frac{\text{pitch}}{\text{base (which is circumf. of cylinder)}}$$

$$\text{or } \sin \angle A = \frac{BC}{AC} \text{ or } = \frac{\text{pitch}}{\text{whole thread}} = \frac{\text{length}}{\text{thread}}$$

and length of screw : pitch :: thread of blade : the whole thread,

$$\text{or length of screw} = \frac{\text{pitch} \times \text{thread of blade}}{\text{whole thread}} \text{ \&c.}$$

Various other formulæ are given in works on the subject, or as derived from extensive experiments and elaborate investigation. For the present the above must be offered herein as mere deductions in aid of practice, and to encourage further examination.

169. For example, an equation is in use for finding what should be the proper pitch of a screw.

$$\text{proper pitch} = \frac{\text{velocity of ship in a minute} \times \text{slip}}{\text{number of intended revolutions in a minute.}}$$

It is obvious that, in every complete revolution of the screw, the vessel would, if the water were a solid, advance through the water a distance equal to the pitch, and therefore the number of revolutions per minute multiplied by this pitch would be the velocity per minute. But if we compare this result with the actual speed as ascertained by heaving of the log, or at the measured mile, we shall find that the speed by calculation is always greater than the

actual measurement. The apparent loss of speed is therefore called the "slip;" and we add the "slip" to the actual velocity of the ship in order to satisfy the following equation: —

Speed = pitch of screw × number of revolutions per minute.

170. This subject is very usefully treated in the "Marine Steam Engine, by Main and Browne," to which the reader is referred for much valuable detail, and especially for its accompanying book of questions on the marine engine. No student of steam should be without it.

The broad-ended blade, as given in the above figure, has fast given place to various modifications, such as the rounding off of the ends, &c.; and, indeed, some screw propellers are now introduced in which the blade does not form part of a real "thread:" this arises from an attempt to diminish slip. But for a few years to come the question of form of screw will be in its infancy.

What is called "negative" slip is caused by the imperfectly formed or defective after-lines of a ship "dragging" water after her in her wake; and if we assume this to be a current *setting in the direction of the ship's* headway, and the screw *working in it*, it is easily perceptible that the ship's actual speed may exceed that by calculation (from the number of revolutions × pitch as above noticed). Engineers, when the vessel is under sail, watch the dynamometer, which indicates the thrust upon the end of the shaft, to see that the sails do not overpower the screw and "drag" it, because, when such is the case, the expenditure of fuel and steam is waste; and the engines may then be thrown out of gear.

SUPERHEATED STEAM.

171. The recent introduction of steam at an increased temperature into use in Her Majesty's navy, is suggestive of the probability of yet further changes being made in the construction of the marine engine. By superheated or surcharged steam, is understood steam which has undergone considerable elevation of temperature after leaving contact with the water surface of the boiler. This is effected by being received into a kind of second steam chest, through which run tubes, which we may call hot pipes; these increase the quantity of steam by converting all the moisture of the ordinary steam into more steam, until, indeed, the temperature is such as to equal that which is able to melt lead, &c. The experiments of Messrs. Penn, upon a large scale, have already resulted in the announcement of a great saving of fuel, and this will most probably be augmented as our experience increases. Many serious considerations, however, attach to the use of superheated steam; for it has been found to destroy in some cases the packing of the piston, &c., in consequence of the increased temperature imparted to the metal of the cylinder: this is averted by using a mixture of common steam with superheated, of about the temperature of 350°. The combination of the two is made by means of two pipes, uniting as near as possible to the slide valves.

As regards calculations, in ordinary practice, on this subject, the finding of the elastic force of such "combined steam," as it is called, is of primary importance. Several formulæ have been produced to elucidate this. Tredgold's has already been given herein (139), but the one in most common use is a modification of this, as under, viz.: —

For steam of a temperature under 212° Fahr.,

$$\text{the elastic force in inches of mercury} = \left(\frac{\text{temperature} + 175}{387}\right)^{7\cdot713}$$

For steam of a temperature above 212° Fahr.,

$$\text{the elastic force in inches of mercury} = \left(\frac{\text{temperature} + 121}{333}\right)^{6\cdot42}$$

As example: Suppose the temperature to be 300° Fahr.

$$\text{The elastic force} = \left(\frac{300 + 121}{333}\right)^{6\cdot42} = \left(\frac{421}{333}\right)^{6\cdot42}$$

By logarithms,

Log of 421 2·624282
„ 333 2·522444

$0\cdot101838 \times 6\cdot42 = \log 0\cdot63579996$

And the log 0·6358 = nat. numb. 4·59 atmospheres.

30

137·70 ins. of mercury.

Atmospheres. lbs. per sq. ft.
Or, 4·59 × 14·75 = 67·7 lbs. pressure per square inch.

KINGSTON'S VALVES.

All openings in the bottom of a steam vessel, except for the waste water pipes, are fitted with Kingston's valves—a contrivance by which, in case of the blowing-out cocks becoming set, the water is prevented from running out of the boiler. The valve itself is merely a conical plug, which, on being pushed out or in, opens or closes the aperture; and this is done easily by hand.

THERMOMETERS (COMPARISON OF).

172. In Europe, three principal methods are used to express the amount of temperature.

Fahrenheit's thermometer divides the temperature between the freezing point and boiling point into 180° (*i.e.* $32° + 180° = 212°$)

Centigrade divides the same space into 100, which it makes the boiling point of water.

Réaumur's divides the same space into 80.

Hence we get these proportions and equations, viz.—

Let F = Fahrenheit C = Centigrade R = Réaumur.

$$\left.\begin{array}{l} 180 : 100 :: (F-32°) : C \\ \text{or, } 9 : 5 :: (F-32°) : C \end{array}\right\} \therefore 9C = 5(F-32°) \text{ and } \begin{cases} C = \dfrac{5(F-32°)}{9} \\ F = \dfrac{9}{5}C + 32° \end{cases}$$

$$\left.\begin{array}{l} 100 : 80 :: C : R \\ \text{or, } 5 : 4 :: C : R \end{array}\right\} \therefore 5R = 4C \text{ and } \begin{cases} R = \dfrac{4C}{5} \\ C = \dfrac{5R}{4} \end{cases}$$

$$\left.\begin{array}{l} 180 : 80 :: F-32° : R \\ \text{or, } 9 : 4 :: F-32° : R \end{array}\right\} \therefore 9R = 4(F-32°) \text{ and } \begin{cases} R = \dfrac{4(F-32°)}{9} \\ F = \dfrac{9}{4}R + 32° \end{cases}$$

The above will sufficiently explain the rules.

Ex. — I wish to convert 60° Centigrade into Fahrenheit.

$$F = \frac{9}{5}C + 32°, \text{ or } F = \frac{9}{5}(60) + 32° = 108 + 32 = 140°$$

SPECIFIC GRAVITY.

173. The weight of a substance compared with its bulk is called specific gravity. If, for instance, a cubic inch of one substance weighs 10 oz., and a cubic inch of

another substance weighs 20 oz., the latter, being the same in bulk, is said to have twice the specific gravity of the former. Distilled water at the temperature of 60° is the unit of comparison with all substances.

Water, however, seldom differs so much in specific gravity as to cause any appreciable error in practice.

The operation of "taking specific gravity" is very simple. The substance is first weighed in the ordinary method, and its weight noted: suppose it to be 250 grains. A horsehair or fine thread is then made to suspend the substance under one scale, so that it dips into water, and if any air-bubbles adhere to it when under water they should be detached, the weight in water being also noted: suppose it to be 150 grains. The simple rule when using distilled water is—"Divide the weight in air by the loss in water," and the quotient is the specific gravity.

So that 250
150
100)250(2·5 = specific gravity.

The rule is thus obtained, viz:—

Let a = weight of substance in air
w = „ „ in water
s = specific gravity of the water
G = specific gravity of the substance.

Then $(a-w) : a :: s : G$

or, $G = \frac{a \times s}{a-w}$ = from the above $\frac{250 \times 1}{250-150} = \frac{250}{100}$

$= \frac{\text{weight in air}}{\text{loss in water}} = 2{\cdot}5.$

This rule will be useful to the practical engineer to enable him readily to detect the value of coal (174), the density of metal, &c.

For, if a piece of cast iron have specific gravity of, say, 7·2, we can easily ascertain the weight of a cubic foot thereof. A cubic foot of water being 62·425 lbs.,

we have 1 : 7·2 :: 62·425 : 449·46 = weight of a cubic foot of the iron.

To find the specific gravity of a body lighter than water. Attach some heavier substance to the body, say a piece of metal.

1. Weigh the body in air.
2. Weigh the sinker in air.
3. „ „ in water.
4. „ compound in water.

Let a = the loss of compound in water,
b = loss of the *sinker* in water,
c = weight of lighter body in air,
s = specific gravity of water,
g = specific gravity of body.

Ex.—Required the specific gravity of a piece of bees' wax (using a piece of gold as a *sinker*).

	Grains.	
Weight of bees' wax in air . .	42·4	= c
„ piece of gold in air . .	77·5	
„ thread (to tie them) in air	·65	
Weight of compound in air . =	120·55	
Weight of compound in water =	69·40	
Loss of compound in water . .	51·15	= a
Weight of gold in air . . .	77·5	
„ „ in water . . .	72·2	
	5·3	= b

Then specific gravity : c :: s : $(a-b)$

$$\text{Or, specific gravity} = \frac{c \times s}{a-b} = \frac{42{\cdot}4 \times 1}{51{\cdot}15 - 5{\cdot}30} = \frac{42{\cdot}4}{45{\cdot}85} = {\cdot}924$$

COAL.

174. Coal, when pure, consists wholly of carbon and the elements of water, viz. oxygen and hydrogen. According to Karsten, some specimens of Newcastle coal are the purest known. He gives the analysis as —

Carbon . . .	84·99
Hydrogen . .	3·23
Oxygen . . .	11·78

We may take the specific gravity 1·2 (water being 1) as a test of such purity.

Although "anthracite" contains a larger percentage of carbon than Newcastle coal (even as much as 97 per cent.), it is less pure, inasmuch as it contains earthy matter, which acts like a flux to the hot bars of the grate, and forms clinker. The specific gravity of anthracite ranges from 1·3 to 1·6. Hence we see specific gravity may be taken as a test of purity. Although Newcastle coal is to be found of a low specific gravity, it has also a range in this respect from 1·2 to 1·5. Newcastle coal contains part of its carbon as bitumen, while anthracite is entirely free from it. The impurities of anthracite, or "Welsh coal" are principally iron, silica, and alumina; hence the earthy residue after burning; while the principal impurity (when there is any) in Newcastle coal consists of bisulphuret of iron (called iron pyrites), known by its *brass-like* appearance. It is the presence of this bisulphuret which renders it dangerous to keep Newcastle coal in a moist state, as the decomposition of this is the cause of spontaneous combustion. The actual presence of bisulphuret of iron may be easily detected in the following manner. A very small piece of it being placed on charcoal, the flame from a candle blown on it to burn off the sulphur, will leave the residue magnetic; this is best

shown by scattering the crushed residue on paper, and passing a magnet along *under* the paper, when the magnetic dust will follow the direction of the magnet. The flame may be blown through a rolled-up paper tube if no blowpipe be at hand.

The author's first leisure will be employed in publishing, for the use of advanced students, a sequel to this book, in which the various properties of steam, and principles connected with the proper consideration of the engine, will be treated with more detail and investigation.

THE END.

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